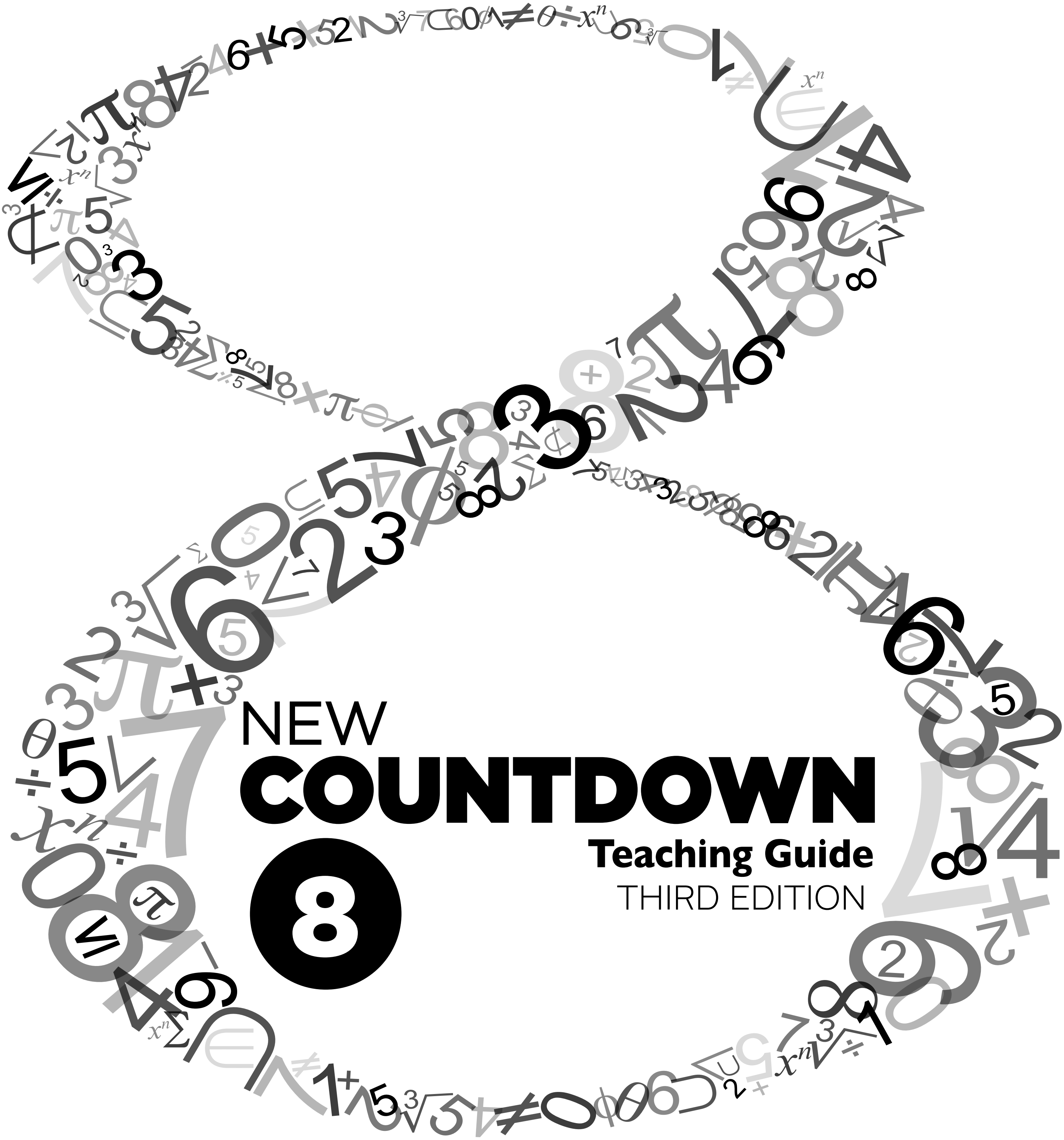

NEW COUNTDOWN

Teaching Guide

8

THIRD EDITION

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CONTENTS

Introduction

Welcome, users of the Countdown series. Countdown has been the choice of Mathematics teachers for many years. This Teaching Guide has been specially designed to help them teach mathematics in the best possible manner. It will serve as a reference book to streamline the teaching and learning experience in the classroom.

Teachers are entrusted with the task of providing support and motivation to their students, especially those who are at the lower end of the spectrum of abilities. In fact, their success is determined by the level of understanding demonstrated by the least able students.

Teachers regulate their efforts and develop a teaching plan that corresponds to the previous knowledge of the students and difficulty of the subject matter. The more well-thought out and comprehensive a teaching plan is, the more effective it is. This teaching guide will help teachers streamline the development of a lesson plan for each topic and guide the teacher on the level of complexity and amount of practice required for each topic. It also helps the teacher introduce effective learning tools to the students to complete their learning process.

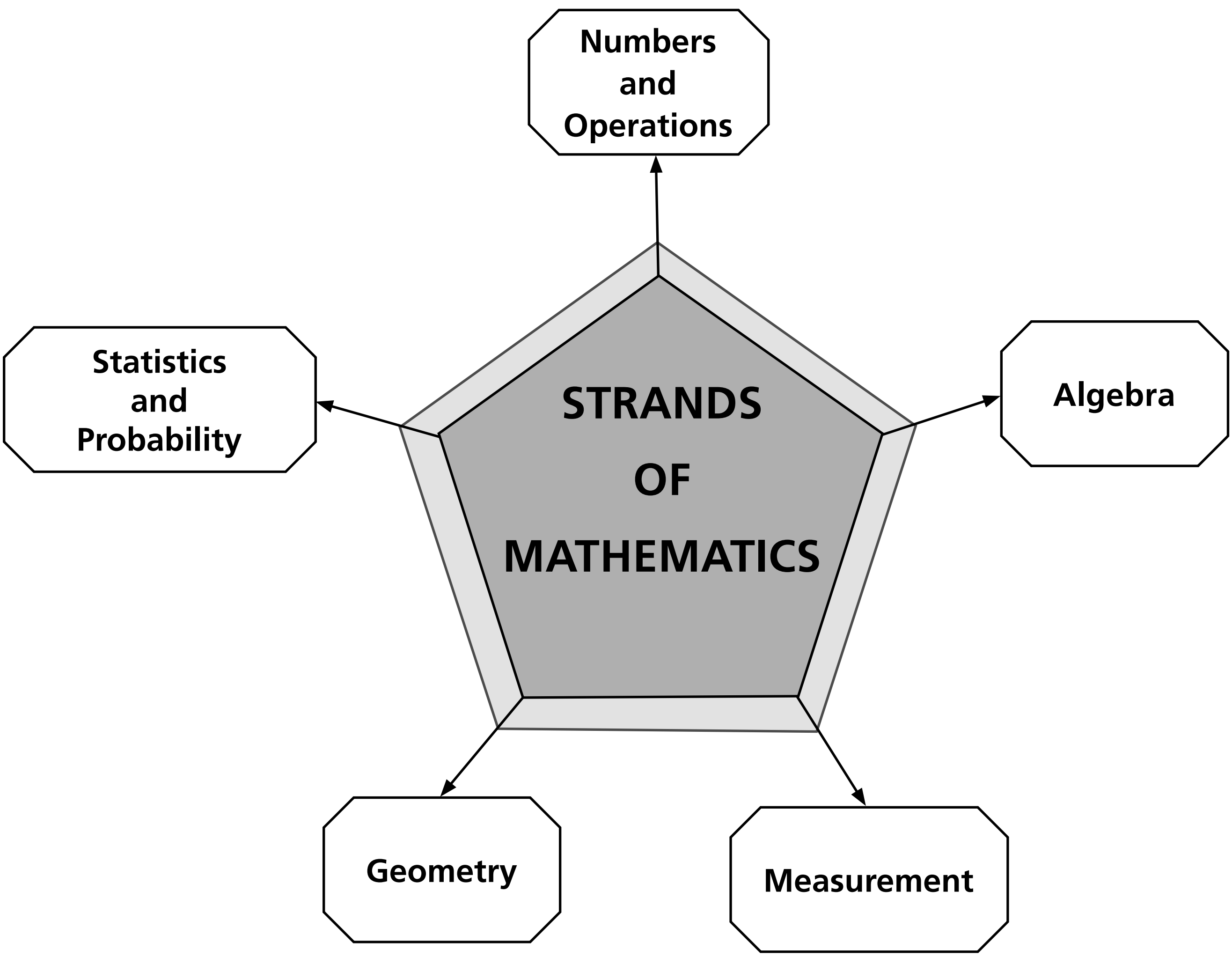
Shazia Asad

Curriculum

Strands and Benchmarks

Pakistan National Curriculum for Mathematics 2022

The Pakistan National Curriculum for Mathematics 2022 is based on these five strands:



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Towards greater focus and coherence of a mathematical programme

A comprehensive and coherent mathematical programme needs to allocate proportional time to all strands. A composite strand covers number, measurement, geometry, algebra, and information handling.

Each strand requires a focussed approach to avoid the pitfall of a broad general approach. If, say, an algebraic strand is approached, coherence and intertwining of concepts within the strand at all grade levels is imperative. The aims and objectives of the grades below and above should be kept in mind.

"What and how students are taught should reflect not only the topics that fall within a certain academic discipline, but also the key ideas that determine how knowledge is organised and generated within that discipline."

William Schmidt and Richard Houang (2002)

Strands and Benchmarks of the Pakistan National Curriculum 2022

Strand	Benchmarks for Grade 6, 7, and 8
Domain A:	**Benchmarks:**
Numbers and Operations	Students will be able use language, notation and Venn diagrams to describe sets and their elements, operate with real numbers, their properties and identify absolute value of real numbers, apply commutative ,associative and distributive laws on real numbers , compare, arrange and round off real numbers to required degree of accuracy, calculate factors, multiples, HCF and LCM, square roots and cube roots, ratio, rate, proportion, percentages, profit, loss, discount, Zakat, Ushr, commission, Taxes, insurance, partnership and Inheritance and apply all of these concepts in real life contexts.
Domain B:	**Benchmarks:**
Algebra	Students will be able to recognise and manipulate number patterns, use letters to represent numbers, expand, simplify, factorise, evaluate and manipulate algebraic expressions, use algebraic identities, interpret and plot graphs of linear equations, solve linear and simultaneous linear equations and linear inequalities and apply all these concepts in real life context.
Domain C:	**Benchmarks:**
Measurement	Students will be able to convert between different units of measure, solve problems involving speed, distance, time, area and perimeter of 2D shapes, surface area and volume of 3D shapes and apply the Pythagorean Theorem.
Domain D:	**Benchmarks:**
Geometry	Students will be able to construct lines, angles of different measure, bisectors of angles, line segments, triangles and quadrilaterals, use the properties of triangles, quadrilaterals, polygons and circles to calculate unknown angles and lengths, apply facts of congruence and similarity and analyse and apply concepts of symmetry and transformations from two and three-dimensional perspectives.
Domain E:	**Benchmarks:**
Statistics and Probability	Students will be able to collect, classify and tabulate statistical data, interpret, construct and use statistical graphs, calculate and interpret measures of central tendency and solve problems using various concepts pertaining to Experimental and Theoretical Probability.

Syllabus Matching Grid

SLOs	Domain A: Numbers and Operations	Covered in NCD 8
M-08-A-01	Round off numbers up to 5 significant figures	Unit 2
M-08-A-02	Analyse approximation error when numbers are rounded off	Unit 2
M-08-A-03	Solve real-world word problems involving approximation	Unit 2
M-08-A-04	Convert Pakistani currency to well-known international currencies and vice versa	Unit 5
M-08-A-05	Differentiate between rational and irrational numbers	Unit 2
M-08-A-06	Represent real numbers on a number line and recognise the absolute value of a real number	Unit 2
M-08-A-07	Demonstrate the ordering properties of real numbers	Unit 2
M-08-A-08	Demonstrate the following properties: - closure property - associative property - existence of identity element - existence of inverses - commutative property - distributive property	Unit 2
M-08-A-09	Solve real-world word problems involving calculation with decimals and fractions	Unit2
M-08-A-10	Identify and differentiate between decimal numbers as terminating (non-recurring) and non-terminating (recurring)	Unit 2
M-08-A-11	Calculate direct and inverse and compound proportion and solve real-world word problems related to direct, inverse and compound proportion. (using table, equation, and graph)	Unit 4
M-08-A-12	Explain and calculate profit percentage, loss percentage, and discount	Unit 5
M-08-A-13	Explain and calculate profit/ mark-up, principal amount and mark-up rate	Unit 5
M-08-A-14	Explain insurance, partnership and inheritance	Unit 5
M-08-A-15	Solve real-world word problems involving profit %, loss %, discount, profit, mark-up, insurance, partnership and inheritance	Unit 5
M-08-A-16	Find the square root of natural numbers, common fractions and decimal numbers (up to 6 digits)	Unit 3
M-08-A-17	Solve real-world word problems involving squares and square roots	Unit 3
M-08-A-18	Recognise perfect cubes and find: - cubes of up to 2-digit numbers - cube roots of up to 5-digit numbers which are perfect cubes	Unit 3
M-08-A-19	Solve real-world word problems involving cubes and cube roots	Unit 3

M-08-A-20	Describe sets using language (tabular, descriptive, and set-builder notation) and Venn diagrams	Unit 1
M-08-A-21	Find the power set (P) of set A where A has up to four elements	Unit 1
M-08-A-22	Describe operations on sets and verify commutative, associative, distributive laws with respect to union and intersection	Unit 1
M-08-A-23	Verify De Morgan's laws and represent through Venn Diagram	Unit 1
M-08-A-24	Apply sets in real-life word problems	Unit 1
SLOs	**Domain B: Algebra**	
M-08-B-01	Differentiate between an arithmetic sequence and a geometric sequence	Unit 7
M-08-B-02	Find terms of an arithmetic sequence using: - term to term rule - position to term rule	Unit 7
M-08-B-03	Construct the formula for the general term (nth term) of an arithmetic sequence	Unit 7
M-08-B-04	Solve real life problems involving number sequences and patterns	Unit 7
M-08-B-05	Recall the difference between: - open and close sentences - expression and equation - equation and inequality	Unit 7
M-08-B-06	Recall the addition and subtraction of polynomials	Unit 7
M-08-B-07	Recall the multiplication of polynomials	Unit 7
M-08-B-08	Divide a polynomial of degree up to 3 by – a monomial – a binomial	Unit 7
M-08-B-09	Simplify algebraic expressions involving addition, subtraction, multiplication and division	Unit 7
M-08-B-10	Recognise the following algebraic identities and use them to expand expressions: $(a + b)^2 = a^2 + b^2 + 2ab$ $(a - b)^2 = a^2 + b^2 - 2ab$ $(a + b)(a - b) = a^2 - b^2$	Unit 8
M-08-B-11	Apply algebraic identities to solve problems like $(103)^2$, $(1.03)^2$, $(99)^2$, 101×99	Unit 8
M-08-B-12	Factorize the following types of expressions: • $ka + kb + kc$ • $ac + ad + bc + bd$ • $a^2 \pm 2ab + b^2$ • $a^2 - b^2$ • $a^2 \pm 2ab + b^2 - c^2$	Unit 8

M-08-B-13	Manipulation of algebraic expressions $(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$ $(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$	Unit 8
M-08-B-14	Construct simultaneous linear equations in two variables	Unit 9
M-08-B-15	Solve simultaneous linear equations in two variables using: - elimination method - substitution method - graphical method division and factorisation method	Unit 9
M-08-B-16	Solve real-world word problems involving two simultaneous linear equations in two variables	Unit 9
M-08-B-17	Identify base, index/ exponent and its value	Unit 6
M-08-B-18	Deduce and apply the following laws of Exponents/ Indices: - Product Law - Quotient Law - Power Law	Unit 6
M-08-B-19	Solve simple linear inequalities that is, $ax > b$ or $cx < d$, $ax + b < c$ $ax + b > c$	Unit 9
M-08-B-20	Represent the solution of linear inequality on the number line	Unit 9
M-08-B-21	Recognise the gradient of a straight line. Recall the equation of horizontal and vertical lines that is, $y = c$ and $x = a$	Unit 9
M-08-B-22	Find the value of 'y' when 'x' is given from the equation and vice versa	Unit 9
M-08-B-23	Plot graphs of linear equations in two variables that is $y = mx$ and $y = mx + c$	Unit 9
M-08-B-24	Interpret the gradient/ slope of the straight line	Unit 9
M-08-B-25	Determine the y-intercept of a straight line	Unit 9
SLOs	**Domain C: Measurement**	
M-08-C-01	State the Pythagoras theorem and use it to solve right-angled triangles	Unit 10
M-08-C-02	Calculate the arc length and the area of the sector of a circle	Unit 10
M-08-C-03	Solve real-life word problems using Pythagoras theorem	Unit 10
M-08-C-04	Calculate the surface area and volume of the pyramid, sphere, hemisphere and cone	Unit 10
M-08-C-05	Solve real-life word problems involving the surface area and volume pyramid, sphere, hemisphere, and cone	Unit 10
SLOs	**Domain D: Geometry**	
M-08-D-01	Rotate an object and find the centre of rotation by construction	Unit 12
M-08-D-02	Enlarge a figure (with the given scale factor) and find the centre and scale factor of enlargement	Unit 12

M-08-D-03	Describe chord, arcs, major and minor arc, semi-circle, segment of a circle, sector, central angle, secant, tangent, and concentric circles	Unit 10
M-08-D-04	Construct a triangle when: -three sides (SSS) - two sides and included angle (SAS) - two angles and included side - a right-angled triangle when hypotenuse and one side (HS) are given	Unit 12
M-08-D-05	Construct different types of quadrilaterals (square, rectangle, parallelogram, trapezium, rhombus and kite).	Unit 12
M-08-D-06	Draw angle and line bisectors to divide angles and sides of triangles and quadrilaterals	Unit 12
M-08-D-07	Identify congruent and similar figures (in your surroundings), apply properties of two figures to be congruent or similar and apply postulates for congruence between triangles	Unit 11
SLOs	**Domain E: Statistics and Probability**	
M-08-E-01	Select and justify the most appropriate graph(s) for a given data set and draw simple conclusions based on the shape of the graph	Unit 13
M-08-E-02	Recognise the difference between discrete, continuous, grouped and ungrouped data	Unit 13
M-08-E-03	Calculate range, variance and standard deviation for ungrouped data and solve related real-world problems	Unit 13
M-08-E-04	Construct frequency distribution tables, histograms (of equal widths) and frequency polygons and solve related real-world problems	Unit 13
M-08-E-05	Explain and compute the probability of; mutually exclusive, independent, simple combined and equally likely events (including real-world word problems	Unit 13
M-08-E-06	Perform probability experiments (for example tossing a coin, rolling a die, spinning a spinner etc. for certain number of times) to estimate probability of a simple event	Unit 13
M-08-E-07	Compare experimental and theoretical probability in simple events	Unit 13

Teaching and Learning

Guiding Principles

1. Students explore mathematical ideas in ways that maintain their enjoyment of and curiosity about mathematics, help them develop depth of understanding, and reflect real-world applications.
2. All students have access to high quality mathematics programmes.
3. Mathematics learning is a lifelong process that begins and continues in the home and extends to school, community settings, and professional life.
4. Mathematics instruction both connects with other disciplines and moves toward integration of mathematical domains.
5. Working together in teams and groups enhances mathematical learning, helps students communicate effectively, and develops social and mathematical skills.
6. Mathematics assessment is a multifaceted tool that monitors student performance, improves instruction, enhances learning, and encourages student self-reflection.

Principle 1

Students explore mathematical ideas in ways that maintain their enjoyment of and curiosity about mathematics, help them develop depth of understanding, and reflect real-world applications.

- The understanding of mathematical concepts depends not only on what is taught, but also hinges on the way the topic is taught.
- In order to plan developmentally appropriate work, it is essential for teachers to familiarise themselves with each individual student's mathematical capacity.
- Students can be encouraged to muse over their learning and express their reasoning through questions such as;
  – *How did you work through this problem?*
  – *Why did you choose this particular strategy to solve the problem?*
  – *Are there other ways? Can you think of them?*
  – *How can you be sure you have the correct solution?*
  – *Could there be more than one correct solution?*
  – *How can you convince me that your solution makes sense?*
- For effective development of mathematical understanding students should undertake tasks of inquiry, reasoning, and problem solving which are similar to real-world experiences.
- Learning is most effective when students are able to establish a connection between the activities within the classroom and real-world experiences.
- Activities, investigations, and projects which facilitate a deeper understanding of mathematics should be strongly encouraged as they promote inquiry, discovery, and mastery.
- Questions for teachers to consider when planning an investigation:
  – *Have I identified and defined the mathematical content of the investigation, activity, or project?*
  – *Have I carefully compared the network of ideas included in the curriculum with the students' knowledge?*
  – *Have I noted discrepancies, misunderstandings, and gaps in students' knowledge as well as evidence of learning?*

Principle 2

All students have access to high quality mathematics programmes.

- Every student should be fairly represented in a classroom and be ensured access to resources.
- Students develop a sense of control of their future if a teacher is attentive to each student's ideas.

Principle 3

Mathematics learning is a lifelong process that begins and continues in the home and extends to school, community settings, and professional life.

- The formation of mathematical ideas is a part of a natural process that accompanies pre-kindergarten students' experience of exploring the world and environment around them. Shape, size, position, and symmetry are ideas that can be understood by playing with toys that can be found in a child's playroom, for example, building blocks.
- Gathering and itemising objects such as stones, shells, toy cars, and erasers, leads to discovery of patterns and classification. At secondary level research data collection, for example, market reviews of the stock market and world economy, is an integral continued learning process. Within the environs of the classroom, projects and assignments can be set which help students relate new concepts to real-life situations.

Principle 4

Mathematics instruction both connects with other disciplines and moves toward integration of mathematical domains.

An evaluation of maths textbooks considered two critical points. The first was, did the textbook include a variety of examples and applications at different levels so that students could proceed from simple to more complex problem-solving situations?

And the second was whether algebra and geometry were truly integrated rather than presented alternately.

- It is important to understand that students are always making connections between their mathematical understanding and other disciplines in addition to the connections with their world.
- An integrated approach to mathematics may include activities which combine sorting, measurement, estimation, and geometry. Such activities should be introduced at primary level.
- At secondary level, connections between algebra and geometry, ideas from discrete mathematics, statistics, and probability, establish connections between mathematics and life at home, at work, and in the community.
- What makes integration efforts successful is open communication between teachers. By observing each other and discussing individual students teachers improve the mathematics programme for students and support their own professional growth.

Principle 5

Working together in teams and groups enhances mathematical learning, helps students communicate effectively, and develops social and mathematical skills.

- The Common Core of Learning suggests that teachers 'develop, test, and evaluate possible solutions'.
- Team work can be beneficial to students in many ways as it encourages them to interact with others and thus enhances self-assessment, exposes them to multiple strategies, and teaches them to be members of a collective workforce.

- Teachers should keep in mind the following considerations when dealing with a group of students:
  – *High expectations and standards should be established for all students, including those with gaps in their knowledge bases.*
  – *Students should be encouraged to achieve their highest potential in mathematics.*
  – *Students learn mathematics at different rates, and the interest of different students' in mathematics varies.*
- Support should be made available to students based on individual needs.
- Levels of mathematics and expectations should be kept high for all students.

Principle 6

Mathematics assessment in the classroom is a multifaceted tool that monitors student performance, improves instruction, enhances learning, and encourages student self-reflection.

- An open-ended assessment facilitates multiple approaches to problems and creative expression of mathematical ideas.
- Portfolio assessments imply that teachers have worked with students to establish individual criteria for selecting work for placement in a portfolio and judging its merit.
- Using observation for assessment purposes serves as a reflection of a students' understanding of mathematics, and the strategies he/she commonly employs to solve problems and his/her learning style.

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Use appropriate tools strategically.
5. Attend to precision and format.
6. Express regularity in repetitive reasoning.
7. Analyse mathematical relationships and use them to solve problems.
8. Apply and extend previous understanding of operations.
9. Use properties of operations to generate equivalent expressions.
10. Investigate, process, develop, and evaluate data.

Lesson Planning

Before starting lesson planning, it is imperative to consider teaching and the art of teaching.

FURL

First **U**nderstand by **R**elating to day-to-day routine, and then **L**earn. It is vital for teachers to relate fine teaching to real-life situations and routine.
'R' is re-teaching and revising, which of course falls under the supplementary/continuity category.
Effective teaching stems from engaging every student in the classroom. This is only possible if you have a comprehensive lesson plan.
There are three integral facets to lesson planning: curriculum, instruction, and evaluation.

1. **Curriculum**

   A syllabus should pertain to the needs of the students and objectives of the school. It should be neither over-ambitious, nor lacking. (One of the major pitfalls in school curricula arises in planning of mathematics.)

2. **Instructions**

Any method of instruction, for example verbal explanation, material aided explanation, or teach-by-asking can be used. The method adopted by the teacher reflects his/her skills. Experience alone does not work, as the most experienced teachers sometime adopt a short-sighted approach; the same could be said for beginner teachers. The best teacher is the one who works out a plan that is customised to the needs of the students, and only such a plan can succeed in achieving the desired objectives.

3. **Evaluation**

The evaluation process should be treated as an integral teaching tool that tells the teachers how effective they have been in their attempt to teach the topic. No evaluation is just a test of student learning; it also assesses how well a teacher has taught.

Evaluation has to be an ongoing process; during the course of study formal teaching should be interspersed with thought-provoking questions, quizzes, assignments, and classwork.

Long-term Lesson Plan

A long-term lesson plan extends over the entire term. Generally schools have coordinators to plan the big picture in the form of Core Syllabus and Unit Studies.
Core syllabi are the topics to be covered during a term. Two things which are very important during planning are the 'Time Frame' and the 'Prerequisites' of the students.
An experienced coordinator will know the depth of the topic and the ability of the students to grasp it in the assigned time frame.

Suggested Unit Study Format

Weeks	Dates	Months	Days	Remarks

Short-term Lesson Planning

A short-term plan is a day-to-day lesson plan, based on the sub-topics chosen from the long-term plan.

Features of the Teaching Guide

The Teaching Guide contains the following features. The headings through which the teachers will be led are explained as follows.

Specific Learning Objectives

Each topic is explained clearly by the author in the textbook with detailed explanation, supported by worked examples. The guide will define and highlight the objectives of the topic. It will also outline the learning outcomes and objectives.

Suggested Time Frame

Timing is important in each of the lesson plans. The guide will provide a suggested time frame. However, every lesson is important in shaping the behavioural and learning patterns of the students. The teacher has the discretion to either extend or shorten the time frame as required.

Prior Knowledge and Revision

It is important to highlight any background knowledge of the topic in question. The guide will identify concepts taught earlier or, in effect, revise the prior knowledge. Revision is essential, otherwise the students may not understand the topic fully.

The initial question when planning for a topic should be how much do the students already know about the topic? If it is an introductory lesson, then a preceding topic could be touched upon, which could lead on to the new topic. In the lesson plan, the teacher can note what prior knowledge the students have of the current topic.

Real-life Application and Activities

Today's students are very proactive. The study of any topic, if not related to practical real-life, will not excite them. Their interest can easily be stimulated if we relate the topic at hand to real-life experiences. Activities and assignments will be suggested which will do just that.

Flash cards based on the concept being taught will have more impact.

Summary of Key Facts

Facts and rules mentioned in the text are listed for quick reference.

Frequently Made Mistakes

It is important to be aware of students' common misunderstandings of certain concepts. If the teacher is aware of these they can be easily rectified during the lessons. Such topical misconceptions are mentioned.

Sample Lesson Plan

Planning your work and then implementing your plan are the building blocks of teaching.

Teachers adopt different teaching methods/approaches to a topic.

A sample lesson plan is provided in every chapter as a preliminary structure that can be followed. A topic is selected and a lesson plan written under the following headings:

Topic

This is the main topic/sub-topic.

Specific Learning Objectives

This identifies the specific learning objective/s of the sub-topic being taught in that particular lesson.

Suggested Duration

Suggested duration is the number of periods required to cover the topic. Generally, class dynamics vary from year to year, so flexibility is important.

The teacher should draw his/her own parameters, but can adjust the teaching time depending on the receptivity of the class to that topic. Note that introduction to a new topic takes longer, but familiar topics tend to take less time.

Key vocabulary

List of mathematical words and terms related to the topic that may need to be pre-taught.

Method and Strategy

This suggests how you could demonstrate, discuss, and explain a topic.

The introduction to the topic can be done through starter activities and recap of previous knowledge which can be linked to the current topic.

Resources (Optional)

This section includes everyday objects and models, exercises given in the chapter, worksheets, assignments, and projects.

Written Assignments

Finally, written assignments can be given for practice. It should be noted that classwork should comprise sums of all levels of difficulty, and once the teacher is sure that students are capable of independent work, homework should be handed out. For continuity, alternate sums from the exercises may be done as classwork and homework.

Supplementary Work (Optional): A project or assignment could be given. It could involve group work or individual research to complement and build on what students have already learnt in class.

The students will do the work at home and may present their findings in class.

Evaluation

At the end of each sub-topic, practice exercises should be done. For further practice, the students can be given a practice worksheet or a comprehensive marked assessment.

1 Operations on Sets

Specific Learning Objectives

In this unit students will learn:

- to describe sets using language (tabular, descriptive, and set-builder notation) and Venn diagrams
- to find the power set (P) of set A where A has up to four elements
- to describe operations on sets and verify commutative, associative, distributive laws with respect to union and intersection
- to verify De Morgan's laws and represent through Venn diagram
- to apply sets in real-world word problems

Suggested Time Frame

6 to 8 periods

Prior Knowledge and Revision

Before starting a chapter it is essential to revise the basic concepts of the topic that were introduced in previous grades. There are various ways of revising. The teacher can initiate a class discussion by asking students to describe and explain concepts they already know. As a further recall technique they can be divided into groups and the board can be divided into two columns. Have a brainstorming revision session by calling students to the board. The teach-by-asking method is a very successful way of revising and also of introducing or explaining a topic.

Students have learnt about sets in previous years and they are aware of the basic concepts of intersection, union, and subsets.

The students should revise complements, union, and intersection, as well as the signs and symbols of subsets. Union is the joining of two sets, where the common and the uncommon elements are written once. A union set is denoted by '$\cup$', which is easy to remember as union begins with 'U'.

Intersection of a set includes the common elements only, which is obvious from the term itself. It is represented by '$\cap$'.

The complement of a set (A′) contains the elements that are not in the universal set. For example, A′ would have all the elements of the universal set that are not members of set A.

Example:

Universal Set = {1, 2, 3, 4, 5, 6, 7, 6, 9, 10}

Set A = {1, 2, 3, 4}

Then Set A′ = {5, 6, 7, 8, 9, 10} (complement of Set A)

An interesting concept: the intersection of a set with its complement will always be a null set.

In the above example:

$$\text{Set } A = \{1, 2, 3, 4\}$$
$$A' = \{5, 6, 7, 8, 9, 10\}$$
$$A \cap A' = \varnothing \text{ or } \{\ \}$$

Similarly the union of a set with its complement will give the universal set, provided that set is the only set of the universal set.

$$\text{Set } A = \{1, 2, 3, 4\}$$
$$A' = \{5, 6, 7, 8, 9, 10\}$$
$$A \cup A' = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$$
$$A \cup A' = \mathbb{U}$$

In this chapter, the concept of the power set is introduced. It tells us how to ascertain the number of subsets that can be made from a given set.

Example:

If A = {1, 2, 3, 4}, then the number of subsets formed will be:

P (A) = 2^k (where k is equal to the number of elements in set A).
$2^4 = 16$ Thus 16 subsets will be formed for Set A.

Exercise 1 contains problems related to power set and combines all previously learnt concepts of sets. It should not be difficult for students to solve the exercise once the concepts have been revised.

Real-life Application and Activities

This chapter at this level becomes quite technical. The key terminology will be of significance to the teacher.

However, the teacher can give a fun quiz where the students can be divided into two groups; one group prepares questions related to the topic and the other group sends its members to answer in turn. Then the activities of the groups are swapped. The group with the most points wins. The teacher can play the role of moderator.

Summary of Key Facts

- A set can be written in three different ways, that is tabular notation, descriptive notation, and set builder notation.
- Set of all the subsets of a set is the power set of that set.
- Two or more sets are called overlapping sets if they have at least one common element.
- Two or more sets are called disjoint sets if they do not have any common element.
- Set A is a subset of a universal set, then the set of elements not in set A is its compliment set.

- Sets can be represented in form of a diagram called a Venn diagram.
- Changing the order of sets in the union operation does not change the answer. This is called the commutative property of union of sets. It states that $A \cup B = B \cup A$.
- The commutative property of intersection of sets, states that: $A \cap B = B \cap A$.
- For three sets A, B, and C the associative property of union of sets, states that: $A \cup (B \cup C) = (A \cup B) \cup C$.
- For three sets A, B, and C the associative property of intersection of sets, states that: $A \cap (B \cap C) = (A \cap B) \cap C$.
- The distributive property of union over intersection of three sets A, B, and C states that: $A \cup (B \cap C) = (A \cup B) \cup (A \cup C)$.
- The distributive property of intersection over union of three sets A, B, and C states that: $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$.
- De Morgan's First law states that: $(A \cup B)' = A' \cap B'$ and Second law states that: $(A \cap B)' = A' \cup B'$.

Frequently Made Mistakes

Students tend to get confused with set notations involving three sets. The first thing to emphasise is that students should be confident in using the signs of unions, intersections, and complements. In order to solve a set notation they should use a Venn diagram.

Sample Lesson Plan

Topic

De Morgan's first law

Specific Learning

Students will able to verify De Morgan's first law.

Suggested Duration

One period

Key Vocabulary

union, intersection, complement

Method and Strategy

De Morgan's first law states: $(A \cup B)' = A' \cap B'$

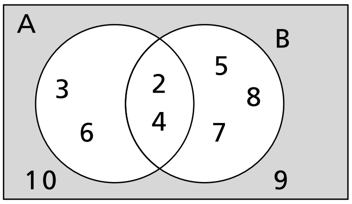
Example:

$\mathbb{U}$ {2, 3, 4, 5, 6, 7, 8, 9, 10}, A = {2, 3, 4, 6}, B = {2, 4, 5, 7, 8}

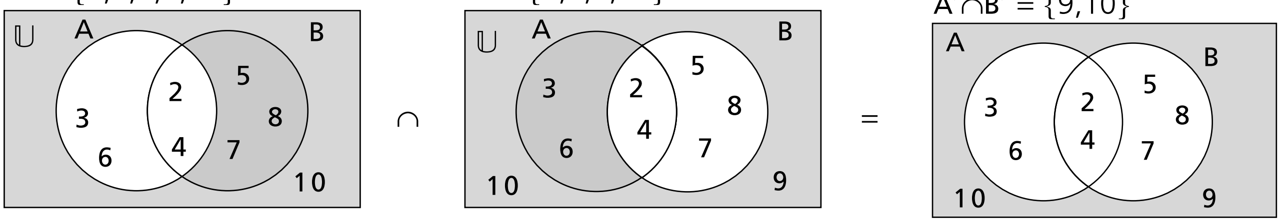
Prove that $((A \cup B)' = A' \cap B'$

To prove that $(A \cup B)' = A' \cap B'$, the left-hand side must be equal to the right-hand side.

$(A \cup B)' = \{9,10\}$



A′ = {5,7,8,9,10} B′ = {3,6,9,10} A′∩B′ = {9,10}



It is important to do these sums on the board by drawing huge Venn diagrams. Using Venn diagrams to solve sets questions is very helpful.

The students can be encouraged to make cutouts of these questions on a sheet of chart paper and display their work on the soft board in the classroom. This will encourage group work and interest in the topic.

Written Assignments

Exercise 1 Q7 and Q8 can be done orally before the activity of making cutouts using the above example. Later they can be done for homework.

Evaluation

Summary points at the end of each lesson will be very helpful as not only will the students revise and strengthen their understanding of key concepts, but also the teacher can use them as a tool to assess learning.

A comprehensive assessment based on multiple choice questions, and advanced conceptual questions similar to the sums in Exercise 1 can also be given.

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2 Real Numbers

Specific Learning Objectives

In this unit students will learn:

- to round off numbers up to 5 significant figures
- to analyse approximation error when numbers are rounded off
- to solve real-world word problems involving approximation
- to differentiate between rational and irrational numbers
- to represent real numbers on a number line and recognise the absolute value of a real number
- to demonstrate the ordering properties of real numbers
- to demonstrate the following properties:
  - closure property
  - associative property
  - existence of identity element
  - existence of inverses
  - commutative property
  - distributive property
- to solve real-world word problems involving calculation with decimals and fractions
- to identify and differentiate between decimal numbers as terminating (non-recurring) and non- terminating (recurring)

Suggested Time Frame

5 to 6 periods

Prior Knowledge and Revision

Students are already aware of rational numbers as taught in earlier classes. They have been introduced to the number line and understand the laws of addition, subtraction, multiplication, and division of rational numbers.

It would be advisable to revise the rules using a number line drawn on the board.

Rational numbers can be expressed in the form of $\frac{p}{q}$, where p and q are integers and $q \neq 0$.

Real-life Application and Activities

The following activity can be done on the board as a fun game.

Divide the students into groups of three.

Write a sum.

Example:

$\left[\frac{1}{4} \div \left(\frac{1}{2} - 2.5 + \frac{1}{4}\right)\right]\pi$

Ask one group to attempt the sum left to right. Ask the next group to follow the order of operation of BODMAS. See who gets the higher value and point out that order of operation matters as they end up with two different answers.

This activity will not only make the students practise together, but will also make them appreciate the significance of BODMAS. Since they will be working in groups they can help each other by pointing out any mistakes and giving the right clue if anyone is unable to grasp the concept.

Ask them to approximate the answer by rounding off to 5 significant figures.

Summary of Key Facts

- Decimals are fractions which have denominators with powers of 10.
- Terminating decimals are decimals that have a finite number of digits after the decimal point.
- Decimal numbers with infinite number of digits after the decimal point are known as non-terminating decimals.
- A rational number can be expressed in the form of $\frac{p}{q}$, where $p, q \in Z$ and $q \neq 0$, whereas an irrational number cannot be expressed in the form of $\frac{p}{q}$.
- Real number is a set that is union of rational and irrational numbers.
- Let a, $b \in \mathbb{R}$, then

  $a \times b = b \times a$

  $a + (b + c) = (a + b) + c$
- Let a, b, $c \in \mathbb{R}$, then

  $a \times (b \times c) = (a \times b) \times c$

  $a \times (b + c) = (a \times b) + (a \times c)$

  and $a \times (b - c) = (a \times b) - (a \times c)$
- If $a \in \mathbb{R}$, then $a + 0 = 0 + a = a$.
- If $a \in \mathbb{R}$, then $a \times 1 = 1 \times a = a$.
- If a, $b \in \mathbb{R}$, and b is the additive inverse of a, then $a + b = b + a = 0$.
- If a, $b \in \mathbb{R}$, and b is the multiplicative inverse of a, then $a \times b = b \times a = 1$.
- Significant figures are the digits in a number that are necessary to indicate the quantity.
- Approximation is a value or quantity that is closer to the exact value.

Frequently Made Mistakes

Students generally get confused with the terminology (or vocabulary) of rational and irrational numbers and their reciprocals. It is important that the earlier terminology (or vocabulary) and concepts of natural, whole numbers and integers are thoroughly revised before the concept of rational numbers is introduced. This is important as this chapter forms the basis of algebra. The students should recognise the significance of the order of operations and the rules of the signs.

Sample Lesson Plan

Topic

Rounding off and approximation

Specific Learning Objectives

Students will be able to rounding off decimals

Suggested Duration

1 period

Key Vocabulary

rounding off, approximation, significant figures

Method and Strategy

A number that needs to be rounded off has to be circled. If the number to its right is 5 or more, then the circled digit is increased by 1 and the rest is deleted.

Example

Round off 28.89 to 2 significant figures.

28. 8 9
    ↑

In other words round off to the nearest whole numbers.

This rounding off can also be explained by using a number line. Whenever a decimal has to be rounded off to the nearest whole number, a number line comes in handy.

A number line can be made on the floor in the corner of the class room and kept during the duration of this chapter. Coloured electrical tape can be taped to the ground to form the number line and the numbers can be placed as flash cards that can be replaced according to the demands of the sums or number sets. The gradings or the dashes on the number line can also be made semi-permanent by making the markings with a different coloured electrical tape.

Written Assignments

The table given in Q3 of Exercise 2B can be done in class. Similar sums can be given for homework.

Evaluation

A comprehensive test can be conducted where learning of all concepts taught can be assessed. Story sums involving rounding off and conversions can be asked. This will develop critical thinking skills.

3 Squares and Square Roots, Cubes, and Cube Roots

Specific Learning Objectives

In this unit students will learn:
- to find the square root of natural numbers, common fractions and decimal numbers (up to 6-digit)
- to solve real-world word problems involving squares and square roots
- to recognise perfect cubes and find:
- cubes of up to 2-digit numbers
- cube roots of up to 5-digit numbers which are perfect cubes
- to solve real-world word problems involving cubes and cube roots

Suggested Time Frame

5 to 6 periods

Prior Knowledge and Revision

The students have learnt about square and cube numbers in earlier grades. A square number is obtained by multiplying the number by itself; cube numbers are obtained when the number is multiplied by itself three times.

An oral quiz of the first ten cube numbers can be done. Flash cards can be made and the students can come one by one, pick up a card, and give its cube.

$1^3 = 1$

$2^3 = 8$

$3^3 = 27$

$4^3 = 64$

$5^3 = 125$

$6^3 = 216$

$7^3 = 343$

$8^3 = 512$

$9^3 = 729$

$10^3 = 1000$

Real-life Application and Activities

This topic may be done as a quiz on the board. The students could be divided into four groups. Each group selects one of its members to go to the board. Prompting by other members of the team may be allowed in the first two rounds, but in the final round, the team member will have to attempt the sum on his/her own.

This is a fun way to learn the steps of mathematical computation by indirect peer participation and instructions.

Perfect squares and square roots can be found by the prime factorisation method.

Example

$$\sqrt{\frac{196}{225}} = \frac{\sqrt{2\times 2} \times \sqrt{7\times 7}}{\sqrt{3\times 3} \times \sqrt{5\times 5}} = \frac{2\times 7}{3\times 5} = \frac{14}{15}$$ [Find prime factor pairs of 196 and 225]

The square root of $\frac{196}{225}$ is $\frac{14}{15}$.

The square roots of numbers that are not square numbers are found by the long division method. Finding the square root of a non-square number is done by making pairs and bringing them down. Once this is done the calculation is the same. In the division method, the first divisor and the new divisor are put together and then used for division.

Example

$$\begin{array}{r|l} & 18.47 \\ \hline 1 & \hat{3}\,\overline{41}.\overline{14}\,\overline{09} \\ +1 & -1 \\ \hline 28 & 241 \\ +8 & -224 \\ \hline 364 & 1714 \\ +4 & -1456 \\ \hline 3687 & 25809 \\ & -25809 \\ \hline & \times \end{array}$$

The rules of the long division method should be revised orally in class so that students know the steps. Displaying these steps on a pin-board during the duration of the chapter would benefit the students.

To find the square roots of decimals the same process is followed but if the decimals are not evenly paired initially, zero is added to complete the pairs.

Decimals can also be converted to fractions and the square roots of the numerator and denominator found separately (either by prime factorisation or long division) and then converted back to decimals as the answer.

Activity

The best way to explain cube numbers is with the use of a 3D object. A square is a 2D shape with equal dimensions; a cube has three dimensions which are equal.

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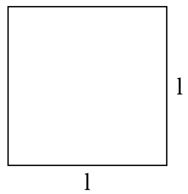
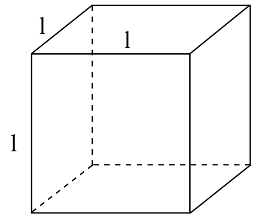


A square has two basic dimensions: $l \times b = l \times l = l^2$



A cube has 3 basic dimensions: $l \times b \times h = l \times l \times l = l^3$

Example

A cube with each side 3 cm has a volume of: $3 \times 3 \times 3 = 27$ cm^3

However, if a solid has a volume of 28 cubic centimetres, then it cannot be a cube because 28 is not a cubic number. It can be a volume of a cuboid: $28 = 2 \times 2 \times 7$.

The teacher should bring cubes and cuboids of different dimensions in class to explain the difference.

The students can make their own presentation of card-board cutouts of cube numbers. The students can be assessed on presentation and understanding. This can be a group or individual marked assignment.

Activity

Recall that the students learned to find LCM by the prime factorisation method in Grade 6. Finding cubes and cube roots involves a similar procedure. Now all they have to do is to make groups of threes instead of groups of twos as they did for square numbers.

Example

Cube root of 729

3	729
3	243
3	81
3	27
3	9
3	3
	1

$\sqrt{\underline{3 \times 3 \times 3} \times \underline{3 \times 3 \times 3}} = \sqrt{3 \times 3} = 9$

Thus 9 is the cube root of 729.

The teacher should also explain that although there are no negative square numbers, cube numbers can be negative because when three negative signs are multiplied the result is negative.

Example

– 8 is a cube number.

$= (-2) \times (-2) \times (-2) = -8$. It is a cube of $(-2)^3$.

Summary of Key Facts

- The square of a number is the product of a number multiplied by itself.
- The square root of a number 'n' is the number that produces the number 'n' when multiplied by itself.
- The cube of a number x is the number x multiplied by itself 3 times.
- For any number n, $\sqrt[3]{n}$ is called the radical, n is the radicand, and 3 is the index of the radical.
- The cube of a negative integer is also negative.
- $\sqrt[3]{mn} = \sqrt[3]{m} \times \sqrt[3]{n}$
- $\sqrt[3]{\frac{p}{q}} = \frac{\sqrt[3]{p}}{\sqrt[3]{q}}$

Frequently Made Mistakes

When doing long division, students generally face problems in finding the clue for the next step. They should write the steps in their notebooks and the teacher should give encouragement and help wherever needed.

Sample Lesson Plan

Topic

Cubes and cube roots

Specific Learning Objectives

Students will be able to make perfect cubes

Suggested Duration

One period

Key Vocabulary Words

smallest possible integer, product, perfect cube

Method and Strategy

"What is the smallest possible value of 24*n*, so that it becomes a perfect cube?"

The teacher can write this question on the board. He/she can explain this concept using the 'teach-by-asking' method.

1. What is a perfect cube?
2. How do we approach this question?
3. Do we break 24 into its prime factors?
4. Which number/numbers are not in sets of three?
5 What is missing to make sets of factors of three?
6. Is this the value of *n*?
7. Is this the smallest possible value of *n*, whereby 24 when multiplied by it makes the number a perfect cube?

If the teaching method is enjoyed, the students should easily grasp this conceptual question.

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Written Assignment

After doing a couple of examples on the board (sums on page 49 of the textbook), students can be given questions to do independently in their notebooks.

What is the smallest integer *'n'* by which the following numbers will become a perfect cube?

	Answers
1. $54n$	4
2. $84n$	882
3. $108n$	2
4. $324n$	18
5. $50n$	20

Evaluation

Give a combined assessment of squares and cubes as this is quite a fun chapter. Questions from Exercises 3A and 3B can be used for the test. Word problems should also be given in the assessment as the students should not treat this chapter in isolation but apply it to real-life situations.

4 Proportions

Specific Learning Objectives

In this unit students will learn:

- to calculate direct and inverse and compound proportion and solve real-world word problems related to direct, inverse and compound proportion. (using table, equation and graph)

Suggested Time Frame

4 to 5 periods

Prior Knowledge and Revision

This chapter is a continuation of the topic of ratios. The teacher should conduct a recall session in which examples of direct and inverse proportions are revised. The facts to be revised are:

- Continued ratios.
- relationship between various quantities

Real-life Application and Activities

Real-life examples can be discussed in class and a brainstorming session can be conducted.

Example

- Number of items and their total cost
- Speed of the car and time
- Number of workers and time taken to complete a certain job
- Number of pipes filling up a tank and the time taken

With the help of these examples, students should be encouraged to explore parity. If one quantity increases, the other also increases. Sometimes if one quantity increases, the other quantity or value decreases.

The difference between direct and inverse proportion should be explained through real-life examples and applications. Only when the students are able to distinguish between the two, should the teacher proceed.

Equations for direct and inverse proportions should be explained. It should be explained that the graphs for both the proportions are different.

Summary of Key Facts

- In direct proportion, an increase in one variable always results in a corresponding increase in other variable.
- If y is directly proportional to x, then the equation expressing y in terms of x is given by, $y = kx$.
- The graph representing direct proportion is always a straight line that passes through the origin.
- In inverse proportion, an increase in one variable always results in a corresponding decrease in the other variable.
- If y is inversely proprtional to x, then the equation expressing y in terms of x is given by, $y = \frac{k}{x}$.
- The graph representing inverse proportion is always a curved line.

Frequently Made Mistakes

This is an easy chapter and the students enjoy it as long as they can differentiate between direct and inverse proportion.

Sample Lesson Plan

Topic

Direct and inverse proportion

Specific Learning Objective

Students will be able to calculate direct and inverse proportion

Suggested Duration

1 period

Key Vocabulary

direct proportion, inverse proportion, curved graph

Method and Strategy

Real-life examples of inverse and direct proportions should be given in class. The teacher should highlight the fact that if one quantity increases, the other decreases, and if one quantity increases, the other increases as well, respectively.

A very logical deduction is that if the speed is greater, the car will take less time to finish a journey.

Activity

A simple activity can be done in class, in which two identical toy cars are brought into the lesson and are pushed with different forces to travel a given distance. The students will record the times on the stop watch and see that more force results in less time, and vice versa.

Once the students have decided the proportion, whether direct or inverse, the teacher should then explain the method. When the operation is inverse, horizontal multiplication is done.

Written Assignment

Questions 1 to 12 of Exercise 4 should be given to the students to carry out the mathematical computation.

Evaluation

A quiz should be given after each concept so that the teacher can assess whether to move on to the next concept or reinforce earlier learning. Quizzes are assessment of learning which are very beneficial.

5 Financial Arithmetic

Specific Learning Objectives

In this unit students will learn:
- to convert Pakistani currency to well-known international currencies and vice versa
- to explain and calculate profit percentage, loss percentage and discount
- to explain and calculate profit/markup, principal amount and markup rate
- to explain insurance, partnership and inheritance
- to solve real world word problems involving profit %, loss %, discount, profit, markup, insurance, partnership and inheritance

Suggested Time Frame

6 to 8 periods

Prior Knowledge and Revision

Students have a good knowledge of percentages. They have done sums on percentages in earlier grades and also used them in earlier chapters. A revision activity can be done.

Activity

The students can be divided into groups of four. They can all make their own sums on percentages and prepare a test paper. Each should contain four sums and the test papers should be swapped between the groups. The test should take ten minutes and the completed answers should be returned to the examiner group. They then check the test and grade the group and hand it back to them. This entire activity should not take more than 25 minutes.

It is a fun revision activity where the students role play teacher, examiner, and students. It is imperative that percentages are revised on the board initially.

Role Play Ativity

The teacher can create a make-believe business and explain the process in the following manner. She/he can even present it on a PowerPoint slide show.

'How business work'

Raheem Brothers buy T-shirts from the suppliers. The cost of buying goods is called the **cost price**.

They then sell the T-shirts to the consumer at a higher price known as the **selling price**.

The difference between the cost price and the selling price is the **profit**.

Loss is incurred if the seller sells at a lower price than the cost price.

To find the profit% or loss% divide the profit or loss by the cost price, and then multiply by 100%.

Example

A shopkeeper buys toy cars in boxes of 50. He purchases 10 boxes for Rs 6000. Each toy car is then sold for Rs 15. Find his profit or loss percentage.

<u>Find</u>

- number of toy cars bought
- cost price of each toy car
- profit made
- profit percentage

$$\text{Cost price} = \frac{6000}{500} = \text{Rs } 12$$

$$\text{Profit} = \text{Rs}15 - \text{Rs}12 = \text{Rs } 3$$

$$\text{Profit\%} = \frac{3}{12} \times 100\% = 25\%$$

Real-life Application and Activities

Percentages can be explained with real-life examples.

Arrange an activity as follows:

Introduce the idea of organising a school concert.

The students role-play organisers. They have to work out the **cost** of the event and then the **profit**. To calculate the cost price:

- calculate the wages of the performers and the support staff
- calculate the total cost of putting on the event e.g: sound, stage, lighting, refreshments, etc.
- profit projection will be based on the number of tickets sold.

Divide students into groups and ask them to work out the total cost of putting on the concert.

Groups are then asked to work out **profit projection** based on three case scenarios.

- 50% of the students attend the concert.
- 75% of the students attend the concert.
- 90% of the students attend the concert.

For each case they calculate the percentage profit using their cost projections and the ticket price.

Lastly, they are asked to calculate the minimum number of tickets they need to sell to break even (no loss is incurred and the cost is covered).

This is an interesting project/assignment and the students can be given two days to complete it. Each group can be marked on this assignment.

Summary of Key Facts

- Percentage profit = $\frac{\text{Profit}}{\text{Cost price}} \times 100\%$
- Percentage loss is calculated as loss in terms of the cost price (CP).
  Percentage loss = $\frac{\text{Loss}}{\text{Cost price}} \times 100\%$
- Discount is a reduction in the marked price of a product.
  Sale price = Marked price – Discount
  Discount = Marked price – Sale price
- Discount = Discount rate × Marked price
- Discount rate = $\frac{\text{Discount}}{\text{Marked price}} \times 100\%$
- When a discount is offered on another discount, it is called a successive discount.
- Insurance is a contractual arrangement in which a person is protected against the risk of loss to his/her property or life.
- Profit = Principal + Markup
- Markup = (Principal × Markup rate per annum × Time in years) ÷ 100
  Markup = (P × R × T) ÷ 100

Frequently Made Mistakes

This topic has real-life applications. Students tend to misunderstand the terms and are unable to apply the correct formula.

Sample Lesson Plan

Topic

Currency conversion

Specific Learning Objectives

Students will be able to convert currency

Suggested Duration

One period

Key Vocabulary

Pounds, US dollars, Rupee, Yen, Riyal, Dirham Currency, exchange rate

Method and Strategy

It will be exciting for the students to investigate and present various currencies of the world. The equivalence rates can be found on the business pages of news papers, or on the internet.

Once the students are familiar with the currencies of different countries, give a short quiz where the teacher names a country and the students name its currency.

Point out the fact that direct proportion is involved in currency conversion.

Currency table for a few countries:

China: Yuan

USA : US dollar

Canada : Canadian dollar

Australia : Australian dollar

Japan : Yen

Pakistan : Rupee

India : Rupee

Bangladesh : Taka

Thailand : Bhat

The list is long and the teacher can add to it.

Written Assignments

The students can copy the list and the teacher can give five sums along with the exchange rates to be calculated.

Evaluation

An objective test based on true or false statements, fill in the blanks, and multiple choice questions can be given on various banking terms.

A comprehensive subjective type of test can also be given at the end of the topic.

6 Algebra: Laws of Indices/Exponents

Specific Learning Objectives

In this unit students will learn:
- to identify base, index/ exponent and its value
- to deduce and apply the following laws of Exponents/Indices:
  - → Product Law
  - → Quotient Law
  - → Power Law

Suggested Time Frame

4 to 5 periods

Prior Knowledge and Revision

Students are familiar with powers and bases. Radical form and surds will be new for them. Hence it is important that the teacher revises the concept of 'the power of'.

Example

a^n

Where a is the base and 'n' the power.

The power or exponent will tell the number (n) of times the base (a) will be multiplied by itself.

Real-life Application and Activities

Activity

The concept of exponential notation can be reintroduced with a fun activity.

You need 'x' packs of cards with the picture cards removed.

Divide the students into groups of four.

Ask one student from each group to deal the cards.

Each student then organises his/her cards in the exponential form.

Example

If the student has 3 fives he will write it as: 5^3

Next ask the students to find the product of their exponential list.

Example

$5^3 = 5 \times 5 \times 5 = 125$, and so on.

Now ask each student to add all the products.

Whoever gets the highest score is the winner. The activity may be timed; the winner is the one who finishes first.

This activity not only develops the students' ability to organise data but is also an indirect way of explaining what exponents / indices / power actually signify.

Summary of Key Facts

- Laws of indices

  Law I: For a non-zero real number a, $a^m \times a^n = a^{m+n}$

  Law II: For a non-zero real number a, $a^m \div a^n = a^{m-n}$

  Law III: For a non-zero real number a, $(a^m)^n = a^{mn}$

  Law IV: For two non-zero real numbers a and b, $(a \times b)^m = a^m \times b^m$

  Law V: For two non-zero real numbers a and b, $\left(\frac{a}{b}\right)^m = a^m \div b^m$

  Law VI: For any non-zero real number a, $a^0 = 1$

- Any number other than zero raised to the power of zero is 1.
- Scientific notation of a number is expressed as $A \times 10^n$, where $1 \leqslant A < 10$ and n is an integer.

Frequently Made Mistakes

Students tend to make mistakes as there is repetitive multiplication. Their oral/mental maths should be sharpened with lots of oral quizzes.

Sample Lesson Plan

Topic

Exponents and radicals

Specific Learning Objective

Students will be able to solve sums of index form

Suggested Duration

One period

Key Vocabulary

index, power, base, exponent

Method and Strategy

The teacher should reinforce the squares and cubes concepts clearly on the board. The students should understand that in 5^2, 5 is the base and 2 is its power / index.

Example

3^4 actually means 3 to the power of 4

$3^4 = 3 \times 3 \times 3 \times 3$.

Example

$5^{-2} \times 5^{-3}$

$= \frac{1}{5^2} \times \frac{1}{5^3}$

$= \frac{1}{5^{2+3}} = \frac{1}{5^5}$

Written Assignments

The following sums can be done in class.

Evaluate:

		Answers
1.	$8 + 4^5$	1032
2.	$2^0 + 4^3$	65
3.	$27^{\frac{1}{3}} + 3^2$	12

Evaluation

This is an important topic. Regular 5 minute quizzes should be given and finally students should be tested with sums that contain all concepts taught.

7 Algebra: Polynomials

Specific Learning Objectives

In this unit students will learn:

- to differentiate between an arithmetic sequence and a geometric sequence
- to find terms of an arithmetic sequence using:
  - → term to term rule
  - → position to term rule
- to construct the formula for the general term (nth term) of an arithmetic sequence
- to solve real life problems involving number sequences and patterns
- to recall the difference between:
  - → open and close sentences
  - → expression and equation
  - → equation and inequality
- to recall the addition and subtraction of polynomials
- to recall the multiplication of polynomials
- to divide a polynomial of degree up to 3 by
  - → a monomial
  - → a binomial
- to simplify algebraic expressions involving addition, subtraction, multiplication and division

Suggested Time Frame

4 to 5 classes

Prior Knowledge and Revision

In Grade 7 students were taught the general term of arithmetic sequences and order, classification, addition, and subtraction of polynomials. It should be pointed out that unlike terms cannot be added or subtracted. These operations can be applied on like terms only. Rules of signs were also explained with the help of a number line.

A revision worksheet can be given. Once completed the teacher can solve the sums on the board and the students can check their partners' work. It is important that each sum is solved on the board as it will be easier for the students to do any corrections. Generally they make a mistake with signs or they tend to add the powers when adding. It should be pointed out that the powers determine whether the terms can be added or subtracted.

Following sums can be given as revision.

1. Add $2x^2 + 3xy$ and $4x^2 - 6xy$.
2. What is the difference between $10wz - 7xz$ and $6wz - 5xz$?
3. What must be added to $3x + 5y$ to get $7x - 3y$?
4. Subtract $2x^2 + 3y - z$ from $-5x^2 + 4y - 2z$.
5. Find the sum of $2x + 3y + z$ and $-5x - 7y - 3z$.

Answers

1. $6x^2 - 3xy$
2. $4x - 2xz$
3. $4x - 8y$
4. $-7x^2 + y - z$
5. $-3x - 4y - 2z$

Real-life Application and Activities

Explain the operations of multiplication and division. These require concrete rules, steps, and method. Addition and subtraction have different rules for signs from to multiplication and division.

The following sign rules are applied in multiplication and division:

(+) × (+) = (+)	(+) ÷ (+) = (+)
(+) × (–) = (–)	(+) ÷ (–) = (–)
(–) × (–) = (+)	(–) ÷ (–) = (+)

Three methods of multiplication are taught in this chapter:

1. vertical multiplication
2. horizontal multiplication
3. the FOIL method

Each method should be explained separately. It is advisable that the same sum is solved by all three methods for comparison. Students will determine the differences in the approaches by remembering the steps involved in each method given on pages 97 and 102.

Activity

Make 20 cards of '+' sign and '–' sign each. Ask two students to come up and play Snap. The teacher will call out any operation, e.g. multiplication or division.

For example, 'multiply two positive signs'.

The students will start looking through the cards one by one.

If he/she get + sign, he/she will call out 'Snap'. Similarly the teacher may say 'divide + sign and – sign'. The student who finds the card with a – sign will says 'Snap'.

Play this game for two minutes, the student with more cards wins.

This activity can be done in pairs till the whole class understands to concept of sign.

Summary of Key Facts

- Arithmetic sequence is advanced by addition or subtraction.
- Geometric sequence is advanced by multiplication or division.
- The formula to construct the nth term formula for any arithmetic sequence is given by $T_n = T_1 + (n-1)\,d$
- $\text{Quotient} = \dfrac{1^{\text{st}} \text{ term of the dividend}}{1^{\text{st}} \text{ term of the divisor}} + \dfrac{\text{Dividend} - 1^{\text{st}} \text{ term of the quotient} \times \text{Divisor}}{\text{Divisor}}$

- FOIL stands for:
  First $\rightarrow$ multiply the first terms
  Outer $\rightarrow$ multiply the outer terms
  Inside $\rightarrow$ multiply the inside terms
  Last $\rightarrow$ multiply the last terms

Frequently Made Mistakes

Students generally get confused with the rules of the signs as they are different from the addition and subtraction rules. Steps and the format for multiplication and division should be written for better understanding.

Sample Lesson Plan

Topic

Operations on polynomials

Specific Learning Objectives

Students will be able to carry out FOIL method of multiplication

Suggested Duration

1 period

Key Vocabulary

FOIL, horizontal multiplication, polynomial

Method and Strategy

In this lesson students will learn a new method of multiplying a binomial by a polynomial.

The teacher will write the acronym 'FOIL' on the board.

First

Outer

Inner

Last

It is advisable that the teacher uses coloured chalk or board markers when explaining this method. The teacher can use a different colour for each letter of FOIL.

Written Assignment

Alternate sums from Exercise 7B can be done for classwork and homework. Students can also solve a sum by using two different methods for practice.

Evaluation

The revision exercise can be used as a format for comprehensive test. Multiplication and division should be tested by specifying the required method in the question. Quizzes should be given at least three times a week in alternate lessons. This 5-minute activity helps the student and teacher identify any problems and then work on them.

8 Algebra: Factorisation and Expansion

Specific Learning Objectives

In this unit students will learn:

- to recognise the following algebraic identities and use them to expand expressions:
  - → $(a + b)^2 = a^2 + b^2 + 2ab$
  - → $(a - b)^2 = a^2 + b^2 - 2ab$
  - → $(a + b)(a - b) = a^2 - b^2$
- to apply algebraic identities to solve problems like $(103)^2$, $(1.03)^2$, $(99)^2$, 101×99
- to factorise the following types of expressions:
  - → $(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$
  - → $(a - b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$
- to do manipulation of algebraic expressions
  - → $(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$
  - → $(a - b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$

Suggested Time Frame

6 to 8 periods

Prior Knowledge and Revision

Students have learnt the three identities in the earlier grade. A quick revision of these identities is extremely important as cubic identities depend on them. The students get confused over when to apply the perfect square or the difference between two squares. In most cases it should be stressed that the sums require initial factorisation and then the application of identities.

The following sums can be given as revision.

	Answers
1. $(2x + 2)^2$	1. $4x^2 + 8x + 4$
2. $(4x - y)^2$	2. $16x^2 - 8xy + y^2$
3. $81x^2 - 1$	3. $(9x + 1)(9x - 1)$
4. $(x + \frac{1}{2}y)^2$	4. $x^2 + xy + y^2$
5. $16x^4 - y^4$	5. $(4x^2 + y^2)(2x - y)(2x + y)$

Explain that perfect squares break up into two squares, and the product of the two terms is multiplied by 2.

The difference between two squares is identified easily as they are individually squared. They are factorised by finding the square root of the terms and then as the sum and difference.

Real-life Application and Activities

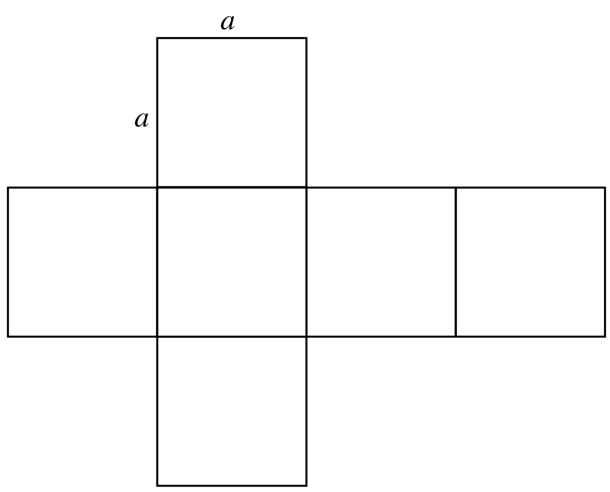
Refer to page 119 of text book where the diagrammatic/geometric explanation of the cubic identity is given.

We can convert it to an activity in class.

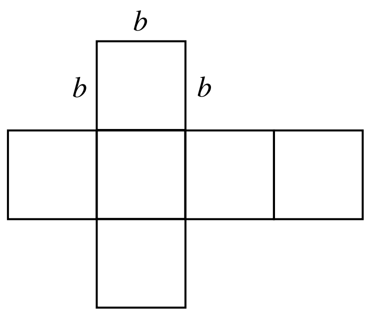
Materials required:

Chart paper with cut out of the net diagrams:

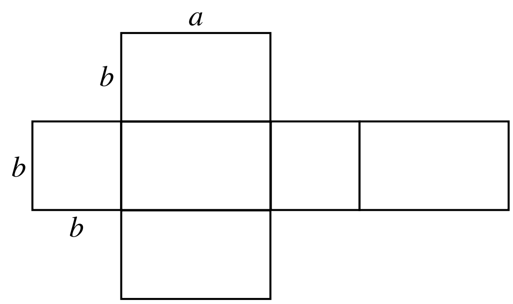
1 cube of side $a \times a \times a$



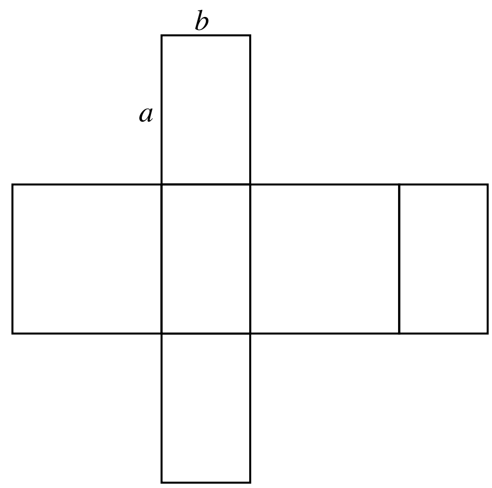
1 cube of side $b \times b \times b$



3 cuboids of side $a \times b \times b$



3 cuboids of side $a \times a \times b$



Once these net diagrams have been made, join them together to make cubes and cuboids.

Ask students to write their individual volumes on each model and combine.

Next step would be to glue all the models and create a big cube of sides $(a+b)^3$.

This branch of algebra where the derivation is done geometrically is called geo-algebra.

This topic is conceptual and lots of examples should be explained on the board, and given to students to solve in their notebooks.

The concepts of cubic identities can be introduced by explaining that this identity is an extension from linear and quadratic functions.

Summary of Key Facts

- Square of the sum of two terms

  $(a + b)^2 = a^2 + 2ab + b^2$
- Square of the difference of two terms

  $(a - b)^2 = a^2 - 2ab + b^2$
- Product of sum and difference of two terms

  $(a + b)(a - b) = a^2 - b^2$
- Cubes of the sum and difference of two terms

  $(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

  $(a - b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$

Frequently Made Mistakes

The sums in this topic will require a lot of concentration as the working of each sum is quite extensive and involves many steps. When doing the sums on the board, the teacher should enunciate each step slowly and carefully. It is common for teachers to keep doing the sums on the board without explaining. If worked carefully, students will memorise, understand the process, and not miss out steps.

Sample Lesson Plan

Topic

Algebraic identities

Specific Learning Objective

Students will be able to resolving cubic terms into factors

Suggested Duration

One to two periods

Key Vocabulary

cubic functions, factorisation

Method and Strategy

Students are already aware of the derivation of the identity, therefore resolving into factors would be easy.

Also students at a time to come on the board and simultaneously do three sums. The rest of the students will prompt any mistakes and also copy the working down. This way each student will get a turn on the board and pear learning will be achieved.

Written Assignments

Exercise 8D, question 5 (parts i to v).

Evaluation

Alternate sums from Exercises 8C and 8D can be given as a test to assess students' ability to use the correct identity.

9 Linear Equations

Specific Learning Objectives

In this unit students will learn:

- to construct simultaneous linear equations in two variables
- to solve simultaneous linear equations in two variables using:
  - → elimination method
  - → substitution method
  - → graphical method division and factorisation method
- to solve real-world word problems involving two simultaneous linear equations in two variables
- to solve simple linear inequalities i.e., $ax > b$ or $cx < d$, $ax + b < c$ or $ax + b >$ c
- to represent the solution of linear inequality on the number line
- to recognise the gradient of a straight line. Recall the equation of horizontal and vertical lines i.e., $y = c$ and $x = a$
- to find the value of 'y' when 'x' is given from the equation and vice versa
- to plot graphs of linear equations in two variables i.e., $y = mx$ and $y = mx + c$
- to interpret the gradient/slope of the straight line
- to determine the y-intercept of a straight line.

Suggested Time Frame

3 to 4 periods

Prior Knowledge and Revision

Students should be able to transpose values in algebraic equations, where the inverse of:

addition is subtraction, subtraction is addition, multiplication is division, and division is multiplication.

They know how to work out the value of an unknown variable 'x'.

A revision exercise can be done on the board, involving students in solving questions step by step.

	Answers
1. $6x + 4 = 7$	1. $\frac{1}{2}$
2. $12x + 4 = 3x + 2$	2. $-\frac{2}{9}$
3. $5x + 7 = 2$	3. -1
4. $22 - x = 4x - 3$	4. 5
5. $35 = 12x - 1$	5. 3

The teacher should point out to the students that they have been solving linear equations with one unknown variable. It is time to move on and solve more complex sums where the linear equations contain two unknown variables.

Generally if the first variable is 'x', then the second unknown variable is taken to be 'y'.

Real-life Application and Activities

This concept can be taught by the 'teach-by-asking' method.

Writes $6x + 4y = 7$ on the board.

Teacher: 'What is new about this equation?'

Students: There are two unknown variables.

Teacher: Good.

To solve two unknowns, we need two equations to solve simultaneously.

So, let us write another equation.

$6x + 4y = 7$ (i)

$3x + y = 2$ (ii)

What should we do now?

Students: Let us eliminate one of the variable.

Teacher: How?

Students: By cancelling 'y'.

Teacher: Correct, but how should we do that?

To eliminate 'y' we need to make:

coefficients $\longrightarrow$ the same

signs $\longrightarrow$ opposite.

How do we make coefficients of 'y' the same?

Students: By making 'y' of equation (ii) become $4y$.

Teacher: 'Great. We can do that by multiplying the entire equation by 4 and then multiplying by –1 to change the signs.'

Hence:

$6x + 4y = 7$ (i)

$3x + y = 2$ (ii)

Multiply equation (ii) by –4 and add both equations.

$$\begin{aligned} 6x + 4y &= 7 \\ \underline{-12x - 4y} &\underline{= -8} \\ -6x &= -1 \\ x &= \frac{1}{6} \end{aligned}$$

Substitute value of x in equation (i).

$$\begin{aligned} 6\left(\frac{1}{6}\right) + 4y &= 7 \\ 1 + 4y &= 7 \\ 4y &= 6 \\ y &= \frac{3}{2} \end{aligned}$$

Answer $\left(\frac{1}{6}, \frac{3}{2}\right)$

Summary of Key Facts

- The gradient of the line tells us how steep the line is or it is a measure of steepness.
- Gradient = $\frac{\text{rise}}{\text{run}}$ or $\frac{\text{vertical change}}{\text{horizontal change}}$ or $\frac{\text{change in } y}{\text{change in } x}$
- Gradient = $\frac{\text{vertical change}}{\text{horizontal change}} = \frac{y_2 - y_1}{x_2 - x_1}$
- The bigger the gradient the steeper the line.
- Inequalities that have the same solution are called equivalent.
- When the two lines are parallel, there is no solution of the two equations.
- When the two lines coincide each other, there is infinite number of solutions.
- y-intercept is the value of y where the line intersects y-axis.
- Two parallel lines have no solution.

Frequently Made Mistakes

When using elimination method the students tend to forget to multiply the entire equation and the constant. The teacher should explain that all the terms on both LHS and RHS of the equation have to be multiplied.

Sample Lesson Plan

Topic

Simultaneous linear equations

Specific Learning Objective

Students will be able to applying real-life conditions and forming simultaneous equations

Suggested Duration

One period

Key Vocabulary

variable, simultaneously

Method and Strategy

When introducing real-life word problems, data representation is extremely critical.

How do you assign the variables 'x' and 'y'?

An entire lesson should be planned on formulating simultaneous linear equations and not solving them. The next lesson can be based on solutions.

Activity

Divide the students into groups of four and write down a sum for each group on the board.

A student from each group will form the equations on the board.

The following sums can be written on the board.

1) Find the other two angles of a right-angled triangle if one angle is $\frac{1}{3}$ of the other.

$x + y = 90°$ (i) (given)

$x = \frac{1}{3}y$ (given)

$\therefore\ 3x - y = 0$ (ii)

Solve equation (i) and (ii) simultaneously

$$\begin{array}{r} x + y = 90° \\ \underline{3x - y = 0} \\ 4x = 90° \end{array}$$

$\therefore\ x = 22.5°$

Substitute the value of x in equation (i)

$x + y = 90°$

$22.5° + y = 90°$

$\therefore\ y = 67.5°$

2) The sum of two numbers is 48 and their difference is 16. Find the two numbers.

$$\begin{array}{r} x + y = 48 \\ x - y = 16 \\ \underline{-\ \ +\ \ \ \ -} \\ x = 32 \end{array}$$

Substitute the value of 'x' in equation (i)

$$32 + y = 48.$$
$$y = 48 - 32$$
$$y = 16$$

3) If aunt is 4 times as old as her nephew and the sum of their present ages is 50, find their present ages.

Suppose aunt's age is x and nephew's age is y

Then $x = 4y$

$x - 4y = 0$ (i)

and $x + y = 50$ (ii)

$$\begin{array}{r} x - 4y = 0 \\ x + y = 50 \\ - \quad - \quad - \\ \hline -5y = -50 \end{array}$$

$$y = 10$$

Substitute the value of 'y' in equation (i)

$$x = 4(10)$$
$$x = 40$$

The aunt is 40 years old and the nephew 10 years old.

Written Assignments

The equations in Questions 6 to 9 of Exercises 9C can be formed in class and solutions done as homework.

Evaluation

Two 5-minute quizzes can be done during this lesson. The first quiz may contain two sums to be solved by the elimination method and the next two sums by the substitution method.

These quizzes will ensure that the students have understood this new concept. This chapter can be taught in three stages: finding a solution by the elimination method, finding a solution by the substitution method, and solving real-life word problems.

A comprehensive test can be given at the end of the chapter. The test should contain sums to solved by the elimination and substitution methods and word problems. Do not specify any method to solve the the word problems. The students should be encouraged to choose their own method to solve them. Generally, students prefer the elimination method to the substitution method.

10 Mensuration

Specific Learning Objectives

In this unit students will learn:

- to state the Pythagoras theorem and use it to solve right angled triangles
- to calculate the arc length and the area of the sector of a circle
- to solve real life word problems using Pythagoras theorem
- to calculate the surface area and volume of the pyramid, sphere, hemisphere, and cone
- to solve real life word problems involving the surface area and volume pyramid, sphere, hemisphere, and cone

Suggested Time Frame

4 to 5 periods

Prior Knowledge and Revision

In earlier classes students have learnt about the area of shapes, volume, and surface area of cubes, cuboids, and cylinders. It will be important to revise all the formulas of each shape on the board. A quick recall quiz can be conducted and the winning group can be awarded marks that can be added to their aggregate marks as a bonus.

Real-life Application and Activities

The students are familiar with the concepts of volume and surface area. The teacher should revise the concepts and explain the link between 2D and 3D figures. It is important to reteach the concept of height/depth to differentiate between 2D and 3D figures.

Cubes and cuboids were studied in grade 7; the formulas could be revised and the students could be given a quiz. This will help the teacher to assess whether or not to move on with the topic.

In this chapter, cylinders, cones, and spheres have been discussed. Before the formulas are introduced, it is imperative that the teacher explains the dimensions of each shape.

Cylinder: this shape has radius and height.

Volume = $\pi r^2 h$

Curved surface area = $2\pi rh + \pi r^2 + \pi r^2$

Total surface area = $2\pi rh + 2\pi r^2$

Cone: this shape has a radius and height, but also slant height 'l'.

Volume: $\frac{1}{3}\pi r^2 h$

Curved surface area = $\pi r l$

Total surface area = $\pi r l + \pi r^2$

Sphere: this shape has a radius and a curved surface.

Volume: $\frac{4}{3}\pi r^3$

Surface area: $4\pi r^2$

If the students understand the formulas and can identify the dimensions, it should not pose a problem for them. Continuous revision of formulas is therefore essentials.

Activity

The students will understand better if the shapes are explained with the help of net diagrams.

To make net diagrams you will need: A4 paper, geometry instruments and markers.

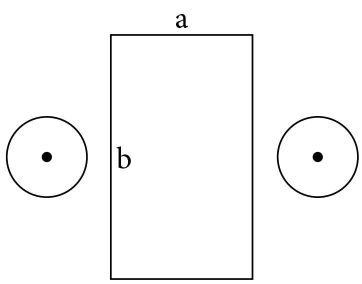
Cylinder: this shape is formed by folding a rectangle into a roll. The breadth of the rectangle becomes the height of the cylinder, and the length, the circumference of its base.

- Net diagram of a cylinder

'a' is the height of the cylinder.

'b' is the circumference of the base of the cylinder.

This activity can be used as a recall in order to understand other shapes better.

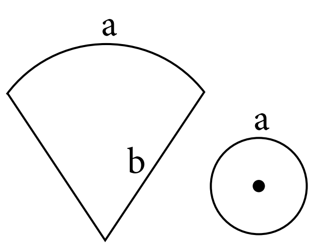


Cones: this shape is formed by folding a semicircle or any fractional part of a circle (sector), and a circle as its base.

- Net diagram of cone

'a' is the circumference of the base of the cone.

'b' is the slant height of the cone.



The students should be encouraged to first cut out the 2D figure and then fold it to form the 3D figure.

Summary of Key Facts

- The circumference of a circle is the boundary or perimeter of the circle.
- A line segment that joins the endpoints of an arc is called a chord.
- Any part of the circumference or perimeter of a circle is known as an arc of the circle.
- Concentric circles are circles with the same centre, but with radii of different lengths.
- Arc length = $\frac{x^\circ}{360} \times 2\pi r$
- Sector Area = $\frac{x^\circ}{360} \times \pi r^2$
- Pythagoras' theorem states that $(\text{hypotenuse})^2 = (\text{base})^2 + (\text{perpendicular})^2$
- The slant height of a pyramid is a line from the vertex to the centre to one edge of the base.
- Volume of a pyramid = $\frac{1}{3}$ × base area × height
- Total surface area of a right circular cone = $\pi rl + \pi r^2$
- Volume of a right circular cone = $\frac{1}{3}$ area of base × height
- Surface area of a sphere = $4\pi r^2$
- Volume of a sphere = $\frac{4}{3}\pi r^3$

Frequently Made Mistakes

Emphasise that the identification of the dimensions are very important. The position of the angle determines the perpendicular and the base. The perpendicular is always opposite to the angle and the base is adjacent to the angle. The hypotenuse is the longest side, opposite the right angle.

Sample Lesson Plan

Topic

Trigonometry

Specific Learning Objective

Students will be able to solve real-life word problems using Pythagoras' Theorem

Suggested Duration

1 to 2 periods

Key Vocabulary

sin, cos, tan, hypotenuse, perpendicular, base

Method and Strategy

Questions 7, 8, 9, 10, and 11 of Exercise 10C are based on real-life applications.

Each sum can be visualised and discussed. Students can give examples of places in school and outside their homes.

If internet is available, real-life videos of places and objects can be shown or five minute PowerPoint presentation by the teacher would help students' understanding.

Once visualised the students can then draw the diagrams of the word problems and apply the Pythagoras' Theorem.

Written Assignments

When each sum has been discussed in detail, students should do the calculations in their notebooks.

Evaluation

A comprehensive assessment should be completed. Time can be assigned to do any corrections as this is also a learning process.

11 Geometry: Congruence and Similarity

Specific Learning Objectives

In this unit students will learn:

- to identify congruent and similar figures (in your surroundings), apply properties of two figures to be congruent or similar and apply postulates for congruence between triangles

Suggested Time Frame

6 to 8 periods

Prior Knowledge and Revision

Students are aware of triangles and other polygons; in this chapter a new concept of congruence and similarity is introduced.

The teacher should brainstorm with the students and prompt and explain the meanings of congruence and similarity.

Congruence: exact same (equal) size, angles, faces etc.

Similarity: same shape but different sizes

The students may come up with geometric instruments of the same brand that are exactly the same and link it with congruence, and of different brands that are similar in relation to sizes.

Real-life Application and Activities

The teacher should explain congruence with real-life examples. Various examples are apartments in building complexes, pots and pans of the exact same size, and Lego blocks. Due to the fact that they are exactly equal in all aspects of measurement, they tend to look like clones. This is a helpful analogy to create and make a list of real-life congruence.

Similarity is best explained with the example of the Russian dolls that fit one inside the other. Though they are of different sizes, they fit inside each other as the curves/angles are the same. The teacher can bring in clay plant pots that are similar and of different sizes, and show that the measurements of length are proportional but the angular aspect remains exactly the same. Similarity is related to enlargement and magnification by a scale factor. Another real-life example could be enlarged pictures on a computer printing each picture in various sizes.

Students should be encouraged to make a table/chart presentation of similar and congruent objects in real-life.

Summary of Key Facts

- Congruent figures have same shape and same size.
- Two shapes are similar if their corresponding angles are equal in measure and all the corresponding sides are in the same ratio.
- Properties of congruent triangles are:

  Side - Side - Side

  Side - Angle - Side

  Angle - Side - Angle

  Angle - Angle - Side

  Right angle - Hypotenuse - Side

Frequently Made Mistakes

The difference between similar and congruent figures must be explained clearly and students must learn the properties. They tend to jumble up the proof of similarity with that of congruence.

Sample Lesson Plan

Topic

Congruence

Specific Learning Objectives

Students will be able to study the cases of RHS and SAS and their application

Suggested Duration

1 period

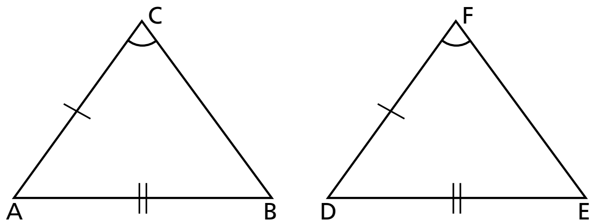
Key Vocabulary

hypotenuse, adjacent and included angle

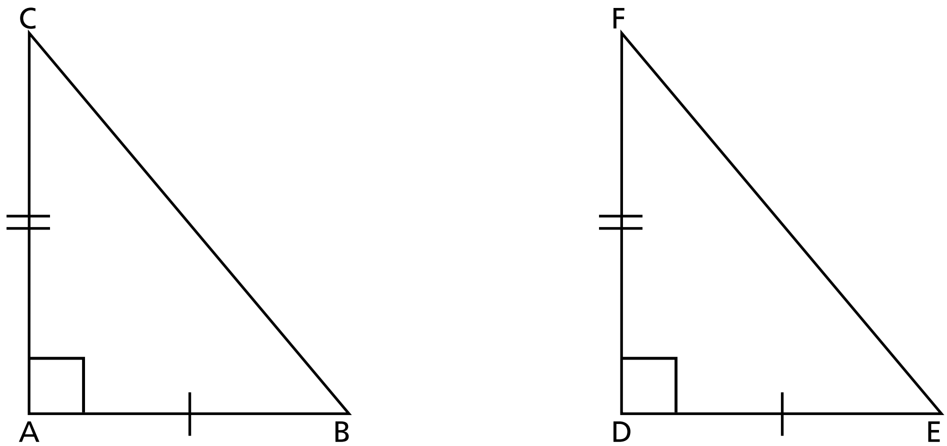
Method and Strategy

It should be made clear to them that when a right-angled triangle has a side and hypotenuse congruent, the case becomes RHS. However, to prove congruency with the included angle should be between the congruent sides. Invariably, a triangle with two sides and one angle which is not in between the two congruent sides makes the case null and void. Similarly, in the case of RHS, two right-angled triangles need not be congruent if two angles and a side, or a right angle with its arms congruent is given. Then, the cases would become ASA and SAS respectively.

Example:



Included ∠A and ∠D are not given, therefore triangles are not congruent.



✗ Not RHS

✓ SAS

Written Assignment

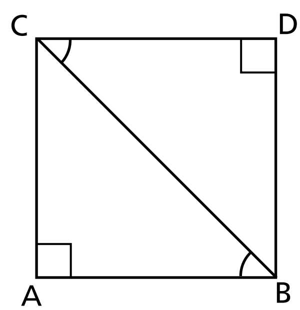
Practice sums from Exercise 11 in the chapter can be done in class and the rest can be given for homework.

The followings sum can be given in class as a quiz.

1. State the case of congruency if congruent.

Answers

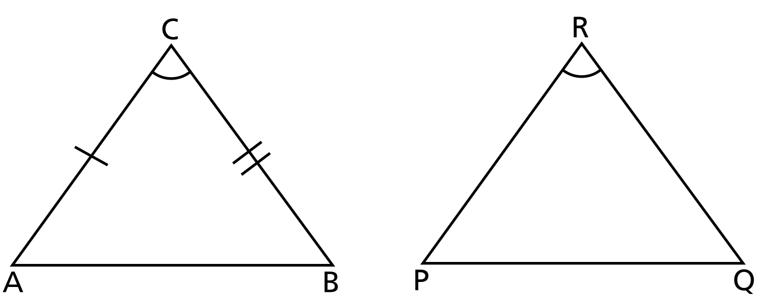
(a)



Yes the triangles are congruent.

Property: AAS

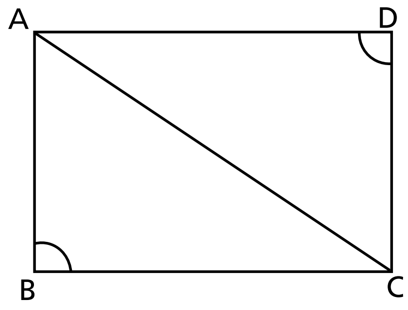
(b)



Yes the triangles are congruent.

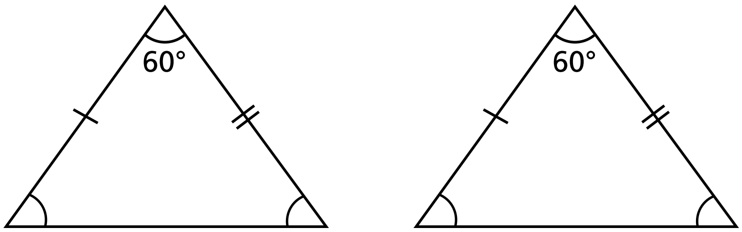
Property: SAS

(c)



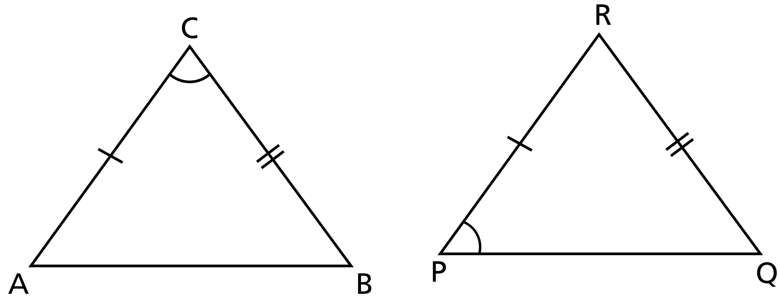
Yes the triangles are congruent.
Property: AAS or SSS

(d)



Yes the triangles are congruent.
Property: SAS

(e)



The triangles are not congruent.
No property is satisfied.

Evaluation

A comprehensive assessment should be given at the end of the topic but in between short quizzes on the board could be conducted towards the end of each lesson to check students' understanding. This chapter introduces an entirely new concept and stage-by-stage assessment is necessary.

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12 Practical Geometry and Transformation

Specific Learning Objectives

In this unit students will learn:

- to rotate an object and find the centre of rotation by construction
- to enlarge a figure (with the given scale factor) and find the centre and scale factor of enlargement
- to construct a triangle when:
  - → three sides (SSS)
  - → two sides and included angle (SAS)
  - → two angles and included side
  - → a right-angled triangle when hypotenuse and one side (HS) are given
- to construct different types of quadrilaterals (square, rectangle, parallelogram, trapezium, rhombus and kite)
- to draw angle and line bisectors to divide angles and sides of triangles and quadrilaterals

Suggested Time Frame

4 to 5 periods

Prior Knowledge and Revision

Students learned the construction of a perpendicular and angle bisectors in the previous grade. It would be advisable to revise the steps on the board and then ask the students to construct two of each type in their notebooks. Revision of construction of triangles with given conditions can also done.

The teacher should do the construction on the board using geometric instruments, as correct handling of the protractor and compasses is important.

Real-life Application and Activities

This chapter requires students to develop skills in handling geometric instruments and remembering the steps of construction.

Once they know all the construction methods a class activity can be done by dividing students into groups. All the different constructions can be written on flash cards. Each group can pick a card and start doing the construction on A4 paper. The group that attempts the most constructions in one period wins.

Summary of Key Facts

- A shape can be rotated around a fixed point. That point is called the centre of rotation.
- An enlargement changes the size of the shape keeping its angles same and sides proportional.
- The point from where the image appears to grow is called its centre of enlargement.
- In enlargement the shapes do not always get larger, but they can get smaller as well.
- The image appears upside down if the scale factor of enlargement is negative.A kite can be constructed when lengths of two side and a diagonal are given.

Frequently Made Mistakes

This chapter is mainly skills-based, and does not require much mathematical reasoning. However, emphasis should be placed on neatness and accuracy. Pencils should be well-sharp, and good compasses should be used which are neither too stiff nor too loose. Students are sometimes unable to draw perfect circum-circles and tangents. The teacher should sort out students' concerns individually.

Sample Lesson Plan

Topic
Geometric constructions

Specific Learning Objective
Students will be able to revise of properties of all shapes

Suggested Duration
1 period

Key vocabulary
bisector, quadrilateral, polygon

Method and Strategy
Parallel lines, bisectors, and the line segments are a few key terms that the students will be dealing with in this chapter. The students will also need to revise the properties of quadrilaterals (rectangle, square, and rhombus). Since they have to do construction, they should be aware of these properties:

Square: all sides are equal and all angles are of 90°

Rectangle: lengths and breadths are equal; all the angles are of 90°.

Rhombus: all sides are equal and the adjacent angles are supplementary.

This chapter not only develops the students' skills in construction, but also encourages them to link mathematical proofs and computation.

Written Assignments

This lesson is primarily a revision class of the properties of shapes and specific construction cases. The students should be encouraged to write all the key points in their notebooks and make a table of facts.

Evaluation

This chapter is a skills-based chapter where ability to construct a clear and precise shape is tested. Mathematical reasoning is also involved as the properties are employed to understand the given construction. There should be regular assessment of learning and understanding and their ability to draw shapes and use mathematical instruments.

13 Data Handling

Specific Learning Objectives

In this unit students will learn:

- to select and justify the most appropriate graph(s) for a given data set and draw simple conclusions based on the shape of the graph
- to recognise the difference between discrete, continuous, grouped and ungrouped data
- to calculate range, variance, and standard deviation for ungrouped data and solve related real-world problems
- to construct frequency distribution tables, histograms (of equal widths), and frequency polygons and solve related real-world problems
- to explain and compute the probability of; mutually exclusive, independent, simple combined and equally likely events. (including real-world problems)
- to perform probability experiments (for example tossing a coin, rolling a dice, spinning a spinner etc. for certain number of times) to estimate probability of a simple event
- to compare experimental and theoretical probability in simple events

Suggested Time Frame

4 to 5 periods

Prior Knowledge and Revision

Students are familiar with this strand of mathematics. Statistical data can be represented in various ways: pictograms, line graphs, pie charts, and bar graphs.

A quick review of this representation can be done on the board where a frequency distribution table is given and the students are asked to represent the data in any two ways of their choice.

Real-life Application and Activities

Activity

Ask the students to collect statistical data and investigate.

For example, students can be given a passage to read and count each of the vowels *a, e, i, o, u.* They can then prepare a questionnaire and collect the data from their friends. Once the data is

collected, the next step is to classify and tabulate. The students can be taught the 'x' and 'y' values as the subject and its frequency respectively. The last stage is the presentation which can be done both as a bar graph and a histogram.
This activity takes a week and at the end their presentations can be put up on the display board.

The students have learnt about statistics. It is based on the collection, organisation, and representation of data. The teacher should give them a revision worksheet of bar graphs where they interpret, read and draw bar graphs.

In this chapter, not only are bar graphs revised, but histograms are also introduced. Interestingly, histograms of grouped data are also introduced. Class intervals or class widths, frequency, and frequency distribution are terms that the students will come across in this chapter.

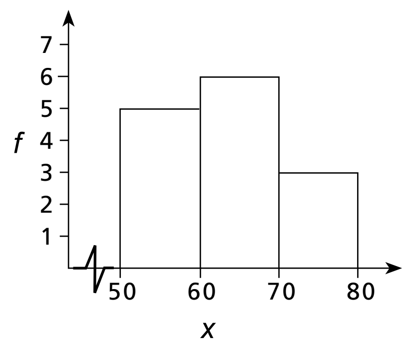
When introducing histograms, the teacher should differentiate clearly between bar graphs and histograms.

The drawing of histograms will also require clear instructions. There are no gaps between the bars of the histograms, and the scale chosen will have to be accurately represented on the graph.

Explain that the 'x' value is sometimes given in the form of groups, and that the values will be written on the graph on the sides of each bar rather than the middle (as is done in bar graphs.)

x	f
$50 < x \leqslant 60$ $60 < x \leqslant 70$ $70 < x \leqslant 80$	5 7 3

A grouped data histogram



'Averages' progresses to 'mean' at this level, as frequency has to be individually multiplied by the value of 'x' to get the 'fx' product.

Example

2, 2, 2, 3, 3, 5, 6, 6, 6

$$\text{Mean} = \Sigma\frac{fx}{f} = \frac{(2)\,(3) + (3)\,(2) + (5)\,(1) + 6\,(3)}{9} = \frac{35}{9} = 3.88 \text{ or } 3.9$$

Activity

This chapter could be used as a project topic where the students can be encouraged to collect data and then represent it on a histogram. The data collected could be:

- number of people investing in four or five types of shares.
- batting statistics of four or five famous batsmen.
- age in years.
- heights or weights of students in class.

Summary of Key Facts

- A frequency distribution table summarizes values and their frequency.
- Histograms are graphical representations of continuous frequency distribution tables.
- A frequency polygon is usually drawn with the help of a histogram.
- Comparison of data is visually more accurate in a frequency polygon graph.
- Probability of an Event = P(E) = $\frac{\text{Number of favourable outcomes}}{\text{Total number of possible outcomes}}$
- When two or more experiments are conducted together or the experiments that involves two or more objects are called combined events.
- The events that do not happen at the same time are called Mutually Exclusive Events.
- When two events are mutually exclusive, add the individual probabilities of both the events. P (A Or B) = P(A) + P(B)
- If the probability of one event does not affect the probability of the other event, the two events are said to be independent.
- If two events are mutually exclusive, they are not independent.
- When two events are independent, multiply the individual probabilities of both the events. P (A And B) = P(A) × P(B)
- In experimental probability, an experiment is performed to gather information.

  P(E) = $\frac{\text{Number of times event occurs}}{\text{Total number of trials}}$

Frequently Made Mistakes

Students generally make mistakes in differentiating between bar graphs and histograms where the first type has intervals or gaps. This is a relatively simple chapter and students tend to enjoy the data presentation.

Sample Lesson Plan

Topic
Data handling

Specific Learning Objective
Students will be able to carry out mode and median

Suggested Duration
One period

Key Vocabulary
median, mode, frequency

Method and Strategy
The students should understand the concept of the mean.

The teacher should then proceed to explain the other measures of central tendency.

Before explaining mode and median, the teacher should explain the most important rule of data collection: to organise the given data in ascending order.

If items of the data have to be grouped or tabulated, the first rule is to arrange them from the smallest to the greatest value.

Once this is done, the mode will be the value that occurs most frequently in a group of data. Therefore, the 'x' value with the highest frequency is the mode.

The median is the middle value, but if the frequency is even-numbered two middle values are selected. If the frequency is odd-numbered then there is only one middle 'x' value.

This concept can be demonstrated with a small activity in class.

Activity

Line up ten students according to their heights in ascending order. The teacher will point out that the measure of their heights are their 'x' value.

The teacher he will then remove one student at a time from the right and the left hand side. In the end, two middle height students will be left. The average of their heights will be the median.

Similarly, the teacher will add on the eleventh student and the process repeated. The students will see that only one student will be left in the middle. The middle value is the median, with equal number of values on either sides.

Written Assignments
Questions 14, 15, and 16 of Exercise 13A can be done in class by the students in their notebooks.

Evaluation
Since this chapter is all about presentation and tabulation, the students can be marked on their classwork and homework assignments. They should be assessed on the accuracy of their presentation and drawings.

A test comprising multiple-choice questions, true and false, and concept-based questions can be given at the end of the chapter.

Assessment

A teacher's journey involves three stages Exposition, Practice, and Consolidation.

Exposition is the setting forth of content, and the quality and extent of the information relayed.

Practice involves problem solving, reasoning and proof, communication, representations, and correction.

Assessment is the final stage of **consolidation** of the process of learning.

Assessment of teaching means taking a measure of its effectiveness.

Assessments

Students can be evaluated on various criteria and by multiple methods (oral or written, projects, tests/ examinations, etc.) during or at the end of a session year.

Assessment is a mandatory part of the teaching and learning process. It cannot be treated isolated from the teaching and learning process. It helps both teachers and learners to judge and evaluate their efforts and pace of learning.

In mathematics it becomes more essential, as mathematical concepts are linked with each other. Concepts grasped during one teaching session serve as a basis for the learning of upcoming concepts. Teachers use assessments for several purposes such as pre-assessing the learners' need, providing relevant instruction, assessing the intended learning outcomes, placement of the learners in different groups, diagnosis of weaknesses and strengths of the learners, adjustment of teaching strategies/ techniques and promotion of the learners to the next grade. Major functions of the assessment are instructional planning, feedback, making decision, and selection of appropriate resources and strategies to move forward. In short, the prime purpose of any assessment is to improve students' learning.

Types of Assessments

Assessment is classified according to its purpose, such as:

- Assessment for Learning (AFL)
- Assessment of Learning (AOL)

Formative Assessments:

These are commonly used as 'assessments for learning.' Formative assessments are conducted throughout teaching practice. They show evidence oo student's learning and are helpful feedback for the teachers to adjust their instructional methods to reduce the learning gaps for students.

In **Assessment for Learning** the teacher provides students with a feedback and support for improvement. The purpose for teachers is to:

- gather evidence of student achievement consistently, fairly, and over short periods of learning time, basically through informal methods
- monitor students' progress towards the defined learning goals
- define teaching adjustments and next steps for teaching to help students reach their potential
- adjust teaching to help students according to their potential

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The most common forms of assessment for learning (formative assessment) are:

In-class activities where students present their findings informally and provide feedback on peer assessments, observations of students non-verbal feedback, homework exercises, questioning (open and closed), quiz, projects, selected responses (may include MCQs, true: false, matching short answers, fill-in-the-blanks, etc), open-ended tasks, performance assessments, process-focused assessments, discussions between student and teacher, answering specific questions, students reflections, students feedback collected through self-assessments etc.

Summative Assessments

These are also known as 'assessments of learning.' Summative assessments check for learners' achievement at the end of the lesson, chapter/ unit, or course. Usually, although not necessarily, these involve formal tests or exams. They are commonly used for grading and ranking students.

Assessment of Learning (Summative)

This assessment leads to the evaluation of student learning. It accurately summarises and communicates to parents, individual students, teachers, other teachers, school leaders and policymakers what students know and can do concerning the overall curriculum expectations.

The teacher assesses a student's summative work at the end of a learning period, to determine to what degree (at what level) the student has achieved the learning goal.

The purpose for teachers is to:

- provide evidence of students' achievement during a specific class and often at the end of a learning unit
- provide assessment data for evaluation
- make judgments about the quality of students learning on the set curriculum expectations
- provide a value (pass/ fail) to that quality of learning achieved by the students
- record and report student's achievements to all stakeholders including parents, teachers, school and senior management as well as students themselves
- use this data as assessment data for the evaluation of student learning

The most common forms of **assessment of learning (summative assessment)** are:

class tests, end of unit tests, monthly tests, mid-year/ annual examinations, standardized tests, multiple choice questions (MCQ), structured papers, presentations (peer or tutor – assessed in controlled environments etc.

Bloom's Cognitive Domains

The cognitive domains given below are used for assessment purpose:

- Knowing: Knowledge
- Applying: Understanding and Application
- Reasoning: Analysis, Synthesis, and Evaluation

Knowing:

Knowing refers to students need to be efficient with the basic knowledge or concept on the recall of mathematical language, basic facts or mathematical concepts, symbolic representation, spatial relations, simple procedures and application of the definitions

Action verbs to knowing are:

- Recall
- Identify
- Interpret
- Describe
- Recognise
- Measure
- Represent
- Explain
- State
- Arrange/ Order

Applying:

Applying refers to students need to be efficient with the application of mathematics in range of contexts. Students need to apply mathematical knowledge of facts, skills and procedures or understanding of mathematical concepts to create representations.

Problem solving is central to applying domain, with an emphasis on more familiar and routine tasks. Problem solving is referred to the real-life problems or concerned with the purely mathematical questions involving numeric or algebraic expressions, functions, equations, geometrical shapes or figures and statistical data sets.

Action Verbs to applying are:

- Examine
- Compute
- Collect
- Differentiate
- Add
- Subtract
- Multiply
- Divide
- Rotate
- Reflect
- Translate
- Enlarge
- Interpret
- Manipulate
- Plot
- Factorise

Reasoning:

Reasoning involves logical and systematic thinking. It includes intuitive and deductive reasoning based on patterns and regularities that can be used to arrive at solutions to problems set in unfamiliar situations. Such problems may be referred to purely mathematical or may have real life settings. For example, the reasoning involves ability to observe and make conjectures. It also involves logical deductions based on specific assumptions and rules.

Action verbs for Reasoning are:

- Analyse
- Predict
- Construct
- Evaluate
- Compare
- Express
- Demonstrate
- Verify
- Solve
- Differentiate

Content Domain

Content domain is the body of knowledge, skills or abilities that are being measured or examined by a test, experiment or research study. It may cover all aspects of the subject area as well as be well-defined objectives.

In secondary level mathematics (Grade VI – VIII), strands and bench marks of the Pakistan National Curriculum (2022) are based on the following content domains:

⇒ Numbers and Operations
⇒ Algebra
⇒ Measurement
⇒ Geometry
⇒ Statistics and Probability

Evaluation

An ideal and fair evaluation involves a plan that is comprehensive. It covers a broad spectrum of all aspects of mathematics. The assessment papers should test every aspect of the topics thought. These can be demarcated into categories: basic, intermediate, and advanced content. The advanced content should be minimal as it tests the most able students only.

Multiple choice questions, also known as fixed choice or selected response items, required students to identify the correct answer from a given set of possible options.

Structured questions assess various aspects of students' understanding: knowledge of content and vocabulary, reasoning skills, and mathematical proofs.

All-in-all the teaching's assessment of students' ability must be based on classroom activity, informal assessment, and final evaluation at the end of a topic and/or the year.

Cognitive domains play vital role in the development of assessment. To assess the student’s in secondary classes the following ratio of cognitive domains are used.

Cognitive Domains/ Skills	Percentage weightage	Comprises of	Covers
Knowing:	20%	Recall	Recall definition, terminology, unit of measurement, geometric shapes and notations
		Describe	Description of numbers, expressions, quantities and shapes by their attributes and properties
		Convert	Conversion of numbers and quantities from one form to another
		Recognise/ Identify	Recognition of numbers, expressions, quantities, shapes and properties
		Arrange/order	Arrange numbers, expressions, quantities and shapes by common properties
		Measures	Measure geometrical shapes, lines, angles and graphs
Applying	40%	Determine	Determine appropriate operations, strategies and tools for solving problems for which there are commonly used methods of solution
		Apply	Application of some rules, algorithm/ formula
		Manipulate	Manipulation of terms, and rules in to simpler form
		Compute	Carry out algorithmic procedure for +, -, x, ÷ or combination of theses with numbers, fractions, decimal and carry out straight forward algebraic expressions
Reasoning	40%	Construct	Construction of tables, geometrical figures and graphs
		Demonstrate	Demonstration of properties of numbers and geometrical figures
		Evaluate	Evaluation of numerical values from expressions, equations, formulas and graphs
		Explain	Explanation of terminologies, formulas, algorithms and properties with reasoning
		Calculate	Calculation of quantities, expressions by using appropriate mathematical operations, formulas and techniques
		Solve	Solution of real-life situations using various mathematical strategies
		Verify	Verification of rules, identities and properties

To develop an assessment tool, a Table of Specification is used to align objectives, instructions and assessment. For example, following table explain weightage of specific topics with respect to different strands in accordance with the curriculum.

Unit Wise Weightage to be used for Table of Specification for Grade VIII

Sr. #	Strand	Title	Weightage	Total	Cognitive Domains/ Skills
1	**Numbers and Operations**	Sets	7%	37%	**K: 20%** **A: 40%** **R: 40%**
2		Real Numbers	6%		
3		Square and Square Roots, Cubes and Cube roots	9%		
4		Variation	4%		
5		Financial Mathematics	11%		
6	**Algebra**	Algebra	11%	18%	**K: 20%** **A: 40%** **R: 40%**
7		Linear Equations	7%		
8	**Geometry**	Geometry	9%	18%	**K: 20%** **A: 40%** **R: 40%**
9		Practical Geometry	5%		
10		Transformations	4%		
11	**Measurement**	Mensuration	9%	9%	**K: 20%** **A: 40%** **R: 40%**
12	**Statistics and Probability**	Data Handling	7%	18%	**K: 20%** **A: 40%** **R: 40%**
13		Probability	11%		
		Total Weightage	100%	100%	

Key: **Knowing (K)** **Applying (A)** **Reasoning: (R)**

[Acknowledgement: Text related to assessment is with reference to Pakistan National Curriculum 2022.]